Pattern Recogn. Phys., 1, 143–146, 2013
www.pattern-recogn-phys.net/1/143/2013/
doi:10.5194/prp-1-143-2013

Pattern Recognition in Physics
Open Access

Celestial commensurabilities: some special cases

H. Jelbring

Tellus, Stockholm, Sweden

Correspondence to: H. Jelbring (hans.jelbring@telia.com)

Received: 5 October 2013 – Revised: 24 October 2013 – Accepted: 30 October 2013 – Published: 2 December 2013

Abstract. Commensurabilities are calculated based on published orbital periods of planets and satellites. Examples are given for commensurabilities that are strong or very strong. There are sets of commensurabilities that involve 3–4 celestial bodies. Our moon–Earth system is probably a key system forming commensurabilities with all the inner planets. The existence and structure of commensurabilities indicate that all celestial bodies in our Solar System interact energetically. The Solar System seems to include an unknown physical process capable of transferring energy between both celestial bodies (orbital energy) and between orbital energy and rotational energy. Such a process is proposed to be the major reason for the evolution of commensurabilities, which are judged as not being produced by chance. The physical reason for their creation still remains undiscovered, however.

1 Background

It is well known that orbital or rotational periods of celestial objects sometimes interlock with each other. The mathematical definition of commensurability is: "*exactly divisible by the same unit an integral number of times*". Our moon always shows the same side towards us. The moon rotates (relative to the stars) at exactly the same period as it orbits around Earth. This is an example of commensurability. As a satellite to a planet it is not alone. Almost all inner satellites to the giant planets behave like our moon does. They turn the same face towards its mother planet at all times.

The concept of commensurability became popular when the Kirkwood gaps among the asteroids were discovered. It turned out that they avoid orbiting at certain (small) rational numbers times the period of Jupiter such as 1/2, 3/7, 2/5 and 1/3. The well-known Bode–Titus law suggests fitting all the planets into approximate commensurabilities (Boeyens, 2009). This "law" does not produce commensurabilities as defined here even if it turns out to be a physical process that can explain the temporal distribution of planetary periods in some approximate way. A similar fair "law" can be found between the inner Jovian satellites, whose orbital periods approximately turn out to be related as 1 : 2 : 4.

A more serious attempt to find commensurabilities among planets was made by Rhodes Fairbridge, who pairwise quantified a number of relationships between planetary orbital periods (see: "Commensurability", "Kirkwood", "Asteroid resonance" and "Orbital commensurability and resonance" in Shirley and Fairbridge, 1997). It remains to find out if there are commensurabilities between orbital periods and rotational periods among all bodies in the Solar System. Allan (1971) indicates (1) that the Newton gravity formula plus Kepler's law based on observations are not enough to predict long-term orbital motion, and (2) that orbital motion is affected by resonances with the rotational period of a planet (in this case Earth).

2 Purpose of article

A number of scientists claim that observed (close) commensurabilities are just produced by chance, while others consider them to be an important result of the Solar System evolution. These commensurabilities should be remarkable enough to warrant further investigation necessary for increased knowledge and understanding of the Solar System. It is the author's opinion that commensurabilities are a result of energy transfer between celestial bodies that have evolved during an extended time period, and that the physical processes responsible are as yet inadequately known. In this paper some known and some unpublished commensurabilities will be presented. The few examples mentioned here will be

Table 1. Orbital periods.

Planets/satellites	years or days	denotation
Mercury	0.2408 87.969	Tme
Venus	0.6152 224.701	Tv
Earth	1.00000	Te
Mars	1.8809	Tma
Jupiter	11.8622	Tj
Saturn	29.4577	Ts
Uranus	84.013	Tu
Neptune	164.79	Tn
Pluto	248.4	Tp
Synodic month	29.53059	Tsyn
Sidereal month	27.32166	Tsid
Anomalistic month	27.55455	Tano
Draconitic month	27.21222	Tdra
Tropical month	27.32158	Ttro
Io	1.769138	Tio
Europa	3.551181	Teu
Ganymede	7.154553	Tga
Callisto	16.689018	Tca
Saros period	6585+1/3	T(Saros)

briefly discussed in the context of Solar System evolution. The intention is to raise awareness of the fact that commensurabilities are not created by chance. The motion of celestial bodies in our Solar System is neither strictly Keplerian nor "chaotic".

3 Method

The orbital periods (Table 1) are mostly quoted from Nordling and Österman (1980). The reason for using planetary orbital periods from this source instead of modern NASA data (2013) is that the former is based on long-term visual records of celestial bodies, while the NASA records are based on short-term instrumental records. Orbital periods in the Solar System are not strictly constant. They vary considerably, but their average values are quite stable over long time scales. However, it is not known whether the long-term planetary periods are quietly diminishing or weakly pulsating. They might even be both at the same time. An examination of commensurabilities provides some answers relating to probable paths of the Solar System evolution. A number of commensurabilities will be calculated below and these will be assigned a simple quality value.

Note that the orbital periods for Mercury and Venus are given to only four significant digits and Jupiter and Saturn to six, according to Nordling and Österman (1980). NASA provides 5–6 digits for the inner planets. The quality concept that will be used in this paper is illustrated by the orbital motions of Venus and Earth. The beat frequency between Venus and Earth is here denoted by Tv || Te (Note: The symbol "||" is used to denote the average beat period between two bodies orbiting the same centre, such that $5 \times \mathrm{Tv} \,\|\, \mathrm{Te} = 7.9938$; $8 \times \mathrm{Te} = 8.00000$ yr). The ratio between these numbers is 1.000776. This level of commensurability is designated as being rather weak and will be called a 3-digit commensurability.

4 Examples of commensurabilities

4.1 The Saros cycle and lunar commensurabilities

$223 \times \mathrm{Tsyn} = 6585.32$; $239 \times \mathrm{Tano} = 6585.54$; $242 \times \mathrm{Tdra} = 6585.36$; $241 \times \mathrm{Tsid} = 6584.52$ (days)

The Saros cycle was discovered in Babylon a couple of hundred years BC. This is an excellent example of 4–5 digit commensurabilities. It should be noted that the position of the moon in relation to the stars (Tsid) only qualifies as a 3-digit commensurability. The motion of the perigee and ascending node of the moon will move in opposite directions in relation to the stars and will meet every 5.99673 yr (average value).

$6 \times \mathrm{Te} = 2191.538$; $1 \times \mathrm{Tdra} \,\|\, \mathrm{Tano} = 2190.344$ (days)

This is a 4-digit commensurability. The question arises if these periods are synchronized to Earth's orbital period just by chance or not. Notice that the observed synchronicity is not exact, and that it should not be expected to be so. The Solar System is a dynamic system which has always changed and which will continue to change. However, if the change is slow, it seems that close-to-perfect commensurabilities can and will evolve.

4.2 Days on Mercury and Venus and Earth's orbital period

Many planetary satellites lock one face towards the planet. Is there evidence that the satellite had been spinning before it got locked up? There are two arguments that should be considered. Most asteroids do rotate, often with a rotation period around 10 h, and then there are the rotation periods of the planets Mercury and Venus, which provide good examples that planets might approach a steady state with a very slow rotation. In this case these periods seem coupled with Earth's orbital period, and it seems a plausible hypothesis that both Mercury and Venus once rotated much faster. According to NASA (2013), Mercury's average day is 175.2 days long and its sidereal rotation period is (on average) 58.65 days. Mercury and Earth are at closest approach every 115.88 (Tme || Te) days, which is the most favourable time for scientists to observe its surface. During such conjunctions, surface visibility is limited by strong sunlight. This is the reason why earlier it was wrongly believed that Mercury's rotation was synchronized with its orbital period. At every third inferior conjunction, Mercury presents the same side towards Earth.

$2 \times 175.2 = 350.4$; $3 \times 115.88 = 347.64$ (days)

This is a weak commensurability, but it seems to indicate an evolution in which Earth is playing a role and which might turn into a strong future commensurability. This suggestion is strengthened by the next example. Venus is slowly rotating in a retrograde direction. There are reasons to believe that Venus has gone from a fast prograde rotation to its current retrograde rotation. Every other planet rotates in a prograde direction (except Uranus, which is a special case). Venus has probably slowed down and then found a "steady state" retrograde rotation, which might be very stable. Venus' rotation period is 243.02 days. Its length of day is 116.75 days.

$6 \times 116.75 = 583.75; \quad 1 \times Tv \| Te = 583.95$ (days)

This is close to a 4-digit commensurability, (very probably) meaning that Earth is affecting the rotation period of Venus in a way that Venus shows the same face towards Earth every time there is an inferior conjunction between the two planets.

4.3 The Galilean satellites

The orbital periods are known to a high precision with 7 digits. A glance at the periods in Table 1 is enough to see that consecutive periods among Jupiter's inner satellites are approximately doubled. The best fit is between Europa and Io, where Teu/Tio = 2.00729, which could be called a week commensurability. However, we are looking for something more interesting. It is possible to find pairwise commensurabilities as follows.

$283 \times Tio = 500.666; \quad 47 \times Teu = 166.906; \quad 7 \times Tga = 50.0818;$
$30 \times Tca = 500.670; \quad 10 \times Tca = 166.890; \quad 3 \times Tca = 50.0671$ (days)

What makes these commensurabilities really intriguing is that there exists a "master" period for all of them, namely around 500.7 days or 1.371 yr. These are 4–5 digit commensurabilities, involving four celestial bodies.

$283 \times Tio = 500.666; \quad 141 \times Teu = 500.818; \quad 70 \times Tga = 500.818$ (days)

$30 \times Tca = 500.670$ (days);

All the Galilean moons seem to be energetically interlocked with each other.

4.4 The Jupiter–Earth–Mars commensurability

There is an undiscovered strong three-body commensurability between our own planet, Mars and Jupiter. The beat frequency between Earth and Mars is 2.1352(0) yr. This is coupled with the orbital period of Jupiter in the following way:

$50 \times Te \| Tma = 106.76(0); \quad 9 \times Tj = 106.760$ (yr)

Such a 5- or 6-digit commensurability poses the question of whether there are relationships that, on average, are very close to being exact over long time periods. Besides, Earth and Mars are involved in another 4-digit commensurability in which Jupiter is left out.

$37 \times Te \| Ma = 79.0025; \quad 79 \times Te = 79.0000$ (yr)

4.5 The Jupiter–Saturn commensurability

$149 \times Tj = 1767.47; \quad 60 \times Ts = 1767.46; \quad 89 Tj \| Ts = 1767.47$ (yr)

This truly remarkable 6-digit commensurability is close to being exact. The orbital periods might be variable, but the commensurabilities should be of a more stable nature than the periods themselves. It is quite possible that this cycle is the Grand Cycle of our Solar System. It might be the periodicity that Jelbring (1996) discussed with respect to Shove's (1955) sunspot records. The longest periods were hard to be precise about for limitations caused by the length of the time series: "If any specific component should pointed out it is the slowly varying one with a 'period' around 1700 yr. Regarding this component Schove's data can hardly be wrong". It should be pointed out that the period in question related to the phase of sunspot numbers from about 300 BC to 1980 AD.

4.6 Commensurabilities among the inner planets

There are good reasons to consider our moon as a planet rather than a satellite. The major argument is that its orbit is more prone to staying in the ecliptic plane rather than the equatorial plane of Earth. Our moon–Earth system might play a crucial role as an "energy transfer gate" between the planets in our Solar System.

$13 \times Tme \| Te = 1506.06; \quad 51 \times Tsid \| Te = 1506.06;$

$38 \times Tsid \| Tme = 1506.06$ (days)

These are 3 interlocked 6-digit commensurabilities and can hardly be considered as "produced" by chance. They simply imply that there has to exist an unknown force affecting energy transfer in the Solar System. Furthermore, there is also a "master" period, which includes the remaining inner planet Venus.

$969 \times Tsid \| Te = 28615.1$ (days or 78.343 yr)
$920 \times Tsid \| Tv = 28615.2$
$722 \times Tsid \| Tme = 28615.2$
$198 \times Tme \| Tv = 28615.4$
$247 \times Tme \| Te = 28615.1$
$49 \times Tv \| Te = 28613.8$

Note that Tsid || Te is equal to the synodic month.

Our moon's importance for energy transfer is probably demonstrated by the fact that the Tsid || Tv provides a higher quality commensurability compared to Tv || Te. It is quite amazing that 6-digit commensurabilities can arise using only 4-digit values for the periods of Mercury and Venus. A probable explanation is that the mean orbital long-term periods of Mercury and Venus are very close to 0.240800 and 0.615200 yr. The corresponding NASA (2013) values are given with 5 and 6 digits as 87.969 days (0.24084 yr) and 224.701 days (0.61519 yr). Values on planetary orbital periods by Nordling and Österman (1980) are preferred, however, for the reasons given above.

5 Discussion

A celestial commensurability is just a pair of numbers. It does not explain anything except a factual relationship between periods which happen to be described by two integers with a good accuracy. By studying commensurabilities like the ones mentioned above, it is quite hard to ignore them as stochastic phenomena. It is possible to test how much these examples deviate from a random result. Such an exercise is not hard to do. It will not be performed here for reasons of time and space. In this paper, I have focused on a few cases of quite amazing commensurabilities, indicating clear deviations from a random distribution.

This paper demonstrates the existence of 3–4 high-quality body commensurabilities among planets, which is an important discovery. This implies that celestial bodies are able to transfer energy between themselves. It also means that the energy transfer is not strictly dependent on distance between the interacting bodies, as has to be the case for interactions based on Newton's gravity formula. There is a lack of a potent theory explaining how this is possible. Some of the examples show that there are reasons to believe in a type of energy transfer between orbital and rotational energy which is unknown or at least not yet well understood, which, by itself, is an important insight. The study of commensurabilities does not provide strict evidence, but points to directions for more complex research efforts. It is easy to get the impression that all the celestial bodies in the Solar System are constantly interacting with each other.

The existence and evolution of Kirkwood gaps in the asteroid belt certainly support such a view. It seems that certain celestial bodies are more active in forming commensurabilities than others. There is little doubt that Jupiter is the major reason for the Kirkwood gaps to evolve. If the sunspot generation is proven to be caused by physical agents outside the surface of the Sun, Jupiter and Saturn would be the main suspects. The examples relating to the inner planets show, that our moon seems to be an important celestial body and that its synodic and sidereal periods are important orbital periods. There have to be identifiable physical reasons for this situation to evolve, however. This issue is discussed separately in Jelbring (2013).

The ultimate task in the context of Solar System evolution, still urgent to resolve, is the identification of the physical mechanisms creating sunspots. Firstly, it is pivotal to prove if sunspots are (mainly) caused by physical agents residing inside our Sun or outside the Sun's surface. Secondly, the present author is persuaded that advanced knowledge about when, how and why commensurabilities evolve will also give an answer to the riddle of which physical processes are responsible for creating sunspots.

Acknowledgements. Many thanks to engineers and scientists around the world producing measurements and making it possible to interpret how nature functions.

Edited by: N.-A. Mörner
Reviewed by: two anonymous referees

References

Allan, R. R.: Commensurable Eccentric Orbits near Critical Inclination, Celestial Mech., 3, 320–330, 1971.

Boeyens, J. C. A.: Commensurability in the Solar System, Unit for Advanced study, University of Pretoria, 2009.

Jelbring, H.: Analysis of sunspot cycle phase variations – based on D. Justin Schove's proxy data, J. Coastal. Res., 17, 363–369, 1996.

Jelbring, H.: Energy transfer in the Solar System, Pattern Recogn. Phys., in press, 2013.

NASA: Fact sheets, http://nssdc.gsfc.nasa.gov/planetary/planetfact.html, last access: November 2013.

Nordling, C. and Österman J.: Physics handbook, Studentlitteratur, Lund, Sweden, 1980.

Schove, D.: The sunspot cycle, 649 BC to AD 1986, J. Geophys. Res., 60, 127–146, 1955.

Shirley, J. H. and Fairbridge, R. W. (Eds.): Encyclopedia of Planetary Sciences, Chapmann & Hall, 1997.