The complex planetary synchronization structure of the solar system

N. Scafetta
Active Cavity Radiometer Irradiance Monitor (ACRIM) Lab, Coronado, CA 92118, USA
Duke University, Durham, NC 27708, USA

Correspondence to: N. Scafetta (nicola.scafetta@gmail.com)

Received: 12 December 2013 – Revised: 19 December 2013 – Accepted: 28 December 2013 – Published: 15 January 2014

Abstract. The complex planetary synchronization structure of the solar system, which since Pythagoras of Samos (ca. 570–495 BC) is known as the music of the spheres, is briefly reviewed from the Renaissance up to contemporary research. Copernicus’ heliocentric model from 1543 suggested that the planets of our solar system form a kind of mutually ordered and quasi-synchronized system. From 1596 to 1619 Kepler formulated preliminary mathematical relations of approximate commensurabilities among the planets, which were later reformulated in the Titius–Bode rule (1766–1772), which successfully predicted the orbital position of Ceres and Uranus. Following the discovery of the ∼11 yr sunspot cycle, in 1859 Wolf suggested that the observed solar variability could be approximately synchronized with the orbital movements of Venus, Earth, Jupiter and Saturn. Modern research has further confirmed that (1) the planetary orbital periods can be approximately deduced from a simple system of resonant frequencies; (2) the solar system oscillates with a specific set of gravitational frequencies, and many of them (e.g., within the range between 3 yr and 100 yr) can be approximately constructed as harmonics of a base period of ∼178.38 yr; and (3) solar and climate records are also characterized by planetary harmonics from the monthly to the millennial timescales. This short review concludes with an emphasis on the contribution of the author’s research on the empirical evidences and physical modeling of both solar and climate variability based on astronomical harmonics. The general conclusion is that the solar system works as a resonator characterized by a specific harmonic planetary structure that also synchronizes the Sun’s activity and the Earth’s climate. The special issue Pattern in solar variability, their planetary origin and terrestrial impacts (Mörner et al., 2013) further develops the ideas about the planetary–solar–terrestrial interaction with the personal contribution of 10 authors.

1 Introduction

In 1543 the De revolutionibus orbium coelestium (On the Revolutions of the Heavenly Spheres) was published. As opposed to Ptolemy’s geocentric model that had been widely accepted since antiquity, Copernicus (1543) proposed a heliocentric model for the solar system: the planets, including the Earth, orbit the Sun and their orbital periods increase with the planetary distance from the Sun. Copernicus also argued that the planets form a kind of mutually ordered system. The physical properties of the planets’ orbits, such as their distances from the Sun and their periods, did not appear to be randomly distributed. They appeared to obey a certain law of nature.

A typical synchronization that could be more easily highlighted by the heliocentric system was, for example, the 8 : 13 Earth–Venus orbital resonance. Every 8 yr the Earth–Venus orbital configuration approximately repeats because the Earth revolves 8 times and Venus ∼13 times, as can be easily calculated using their sidereal orbital periods: \( P_{\text{Es}} = 365.256 \text{ days} \) and \( P_{\text{Ve}} = 224.701 \text{ days} \). Figure 1a demonstrates this orbital regularity by showing the relative positions of Earth and Venus on 1 January from 2012 to 2020.

However, Venus presents a more subtle and remarkable synchronization with Earth. The rotation period of Venus on its own axis is 243.021 days (that is, almost exactly two-thirds of the Earth’s annual period) and is retrograde. It is
easy to calculate that at every inferior conjunction (that is, every time the Sun, Venus and Earth line up), the same side of Venus faces Earth (Goldreich and Peale, 1966a; Jelbring, 2013); the Venus–Earth synodic period is 583.924 days and there are five inferior conjunctions in 8 yr. In fact, as Fig. 1b shows, in one synodic period Earth revolves 1.59867 times around the Sun, while Venus rotates on its own axis 2.40277 times in the opposite direction. The sum of the fractional part of the two numbers is almost exactly 1 (≈ 1.00144). Thus, not only is Earth almost synchronized with Venus in a 8:13 orbital resonance and in a 8:5 synodic resonance but, despite the large distance separating the two planets, it seems to have also synchronized Venus’ rotation. It is unlikely that this phenomenon is just a coincidence.

Earth always sees the same face of the Moon. The lunar rotation has been synchronized with Earth by tidal torque. At least 34 moons of the solar system (e.g., the Galilean moons of Jupiter) are rotationally synchronized with their planet (http://en.wikipedia.org/wiki/Synchronous_rotation). Charon and Pluto are also gravitationally locked and keep the same face toward each other. Mercury’s rotation period (58.646 days) is exactly 2/3 of its orbital period (87.969 days) (Goldreich and Peale, 1966b; Jelbring, 2013). The synchronization of Mercury’s rotation with its orbital period may be due to the combined effect of the strong tidal torque by the Sun and to Mercury’s eccentricity (∼ 0.2), which implies that at perihelion Mercury is about 2/3 of its aphelion distance from the Sun: 0.307 AU versus 0.467 AU. It is also well known that the three inner moons of Jupiter – Ganymede, Europa and Io – participate in a 1:2:4 orbital resonance. However, the synchronous rotation of Venus with the Earth’s orbit is surprising, given the large distance between the two planets. In fact, the theoretical tidal elongation caused by the Earth’s gravity on Venus is just a fraction of millimeter. At the inferior conjunction the tidal elongation caused by Earth on Venus is maximum and is about \(3m_{\text{Ea}}R_{\text{Ve}}^2/2m_{\text{Ve}}d_{\text{VE}}^3 = 0.035 \text{ mm}\), where \(m_{\text{Ea}} = 1\) and \(m_{\text{Ve}} = 0.815\) are the masses of Earth and Venus in Earth’s mass unit, \(R_{\text{Ve}} = 6051.8 \text{ km}\) is the radius of Venus and \(d_{\text{VE}} = 41.4 \times 10^6 \text{ km}\) is the average distance between Earth and Venus at the inferior conjunction.

Numerous other examples of strong commensurabilities among the planets of the solar system have been found, and some of them will be discussed in this paper (cf. Jelbring, 2013; Tattersall, 2013). Furthermore, the 27.3 days sidereal orbital period of the Moon around Earth appears well synchronized with the 27.3 days period of the Carrington rotation of the Sun, as seen from the Earth, which determines a main electromagnetic oscillation of the heliospheric current sheet in a Parker spiral. The collective synchronization among all celestial bodies in our solar system indicates that they interact energetically with each other and have reached a quasi-synchronized dynamical state.

Indeed, the bodies of the solar system interact with each other gravitationally and electromagnetically, and their orbits and rotations are periodic oscillators. As discovered by Christian Huygens in the 17th century, entrainment or synchronization between coupled oscillators requires very little energy exchange if enough time is allowed. Huygens patented the first pendulum clock and first noted that, if hung on the same wall, after a while, pendulum clocks synchronize

Figure 1. (A) Earth and Venus’ orbits and their positions on 1 January for the years 2012 to 2020 in Copernicus’ heliocentric system. The figure shows that every 8 yr the Venus–Earth configuration approximately repeats forming eight-point star pattern. (B) Earth–Venus inferior conjunctions from 2012 to 2020. The figure shows a five-point star pattern. Note that at every conjunction, the same side of Venus (represented by a small cyan circle) faces Earth. The orbits and the coordinates (in astronomical units) of the planets were determined using the JPL’s HORIZONS Ephemeris system (http://ssd.jpl.nasa.gov/horizons.cgi).
to each other due to the weak physical coupling induced by small harmonic vibrations propagating in the wall (Pikovsky, 2001). Note that the solar system is about 5 billion years old, is not part of a stellar binary system, and in its history has not experienced particularly disrupting events such as collisions with other solar systems. Therefore, a certain degree of harmonic synchronization among its components should be expected.

Newtonian mechanics calculates that the theoretical tidal elongation induced by the gravity of the planets inside the Sun is just a fraction of millimeter (Scafetta, 2012c). Therefore, tidal forcing appears too small to effect the Sun. However, as discussed above, the magnitude of the tidal elongation induced by the Earth’s gravity on Venus is also a fraction of millimeter. Thus, if the Earth’s gravity or some other planetary mechanism has synchronized the rotation of Venus with Earth, the planets could have synchronized the internal dynamics of the Sun, and therefore they could be modulating solar activity. It seems simply unlikely that in a solar system where everything appears more or less synchronized with everything else, only the Sun should not be synchronized in some complex way with planetary motion.

Thus, the Earth’s climate could be modulated by a complex harmonic forcing consisting of (1) lunar tidal oscillations acting mostly in the ocean; (2) planetary-induced solar luminosity and electromagnetic oscillations modulating mostly the cloud cover, and therefore the Earth’s albedo; and (3) a gravitational synchronization with the Moon and other planets of the solar system modulating, for example, the Earth’s orbital trajectory and its length of day (cf. Mörner, 2013).

From Kepler’s basic concepts forward through time, this paper briefly summarizes some of the results that have further suggested the existence of a complex synchronization structure permeating the entire solar system whose physical origin is still not fully understood. A number of empirical studies have shown that a complex synchronized planetary harmonic order may characterize not only the solar planetary system but also the Sun’s activity and the Earth’s climate, fully confirming Kepler’s vision about the existence of a harmony of the world. Preliminary physical mechanisms are being proposed as well.

This brief review is not fully comprehensive of all the results. It simply introduces a general reader to this fascinating issue. The next sections review general results found in the scientific literature showing and discussing (1) the ordered structure of the planetary system; (2) the likely planetary origin of the variability of the Sun’s activity; and (3) the synchronization of the Earth’s climate with lunar, planetary and solar harmonics.

2 Kepler’s vision of a cosmographic mystery

About half of a century after Copernicus, Kepler corrected and extended the heliocentric model. Kepler found that (1) the orbit of every planet is an ellipse (instead of Copernicus’ perfect circles) with the Sun at one of the two foci (instead of being in the center of the cycle), (2) a line joining a planet and the Sun sweeps out equal areas during equal intervals of time, and (3) the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit. If the orbital period, \( T \), is measured in years and the semi-major axis, \( a \), is measured in astronomical units (AU, the average Sun–Earth distance), Kepler’s third law takes the simple form of \( T^2 = a^3 \). The first two laws were published in 1609 (Kepler, 1609), while the third law was published in 1619 (Kepler, 1619). Kepler’s three laws of planetary motion were later formally demonstrated by Newton (1687) using calculus and his law of universal gravitation stating that a planet is attracted by the Sun with a force directly proportional to the product of the two masses and inversely proportional to the square of the Sun–planet distance.

However, Kepler did more than just proposing his three laws of planetary motion. Since the publication of the Mysterium Cosmographicum (The Cosmographic Mystery) Kepler (1596) noted the existence of a “marvelous proportion of the celestial spheres” referring to the “number, magnitude, and periodic motions of the heavens”. Kepler found specific distance relationships between the six planets known at that time (Mercury, Venus, Earth, Mars, Jupiter and Saturn). These relationships could be understood in terms of the five Platonic solids enclosed within each other, with the outer solid being a sphere that represented the orbit of Saturn (see Fig. 2a and b).

Some of these geometrical relations are easy to notice. For example, the ratio between the Earth’s orbital radius \( a = 1 \) AU and Venus’ orbital radius \( a = 0.72 \) AU is approximately equal to the ratio between the diagonal and the side of a square (\( \sqrt{2} \approx 1.41 \)). Thus, Venus’ orbit is approximately enclosed within a square enclosed within the Earth’s orbit (see Fig. 1b). Analogously, the ratio between Saturn’s orbital radius \( a = 9.6 \) AU and Jupiter’s orbital radius \( a = 5.2 \) AU is approximately equivalent to the ratio between the diagonal and the side of a cube (\( \sqrt[3]{3} \approx 1.73 \)). Thus, Jupiter’s orbit is approximately enclosed within a cube enclosed within Saturn’s orbital sphere (see Fig. 2a).

Kepler also highlighted the existence of a 5 : 2 Jupiter–Saturn resonance, which had been, however, well known since antiquity (Ma’Sr, 9th century; Temple, 1998): every \( \approx 60 \) yr the Jupiter–Saturn configuration approximately repeats because Jupiter revolves \( \approx 5 \) times and Saturn \( \approx 2 \) times. Figure 2c shows Kepler’s original diagram of the great conjunctions of Saturn and Jupiter, which occur every \( \approx 20 \) yr, from 1583 to 1723. Every three conjunctions (a trigon) Jupiter and Saturn meet approximately at the same location of the zodiac, which happens every \( \approx 60 \) yr. The
trigon slightly rotates and the configuration repeats every 800–1000 yr.

The discovery of a geometrical relationship among the semi-major axes of the planets and the relationship between the planets’ orbital semi-major axis and their orbital period (the third law of planetary motion) convinced Kepler (1619) that the planetary orbits are mutually synchronized as though the solar system formed a kind of celestial choir. The great advantage of the heliocentric model was mostly to make it far easier to see this ordered structure.

Kepler also conjectured that celestial harmonics could permeate the entire solar system, including the Earth’s climate (Kepler, 1601, 1606, 1619). However, modern physics would require that for the planets to modulate the Earth’s climate, they first need to modulate the Sun’s activity. In fact, the Sun is the most likely place where the weak planetary harmonics could be energetically amplified by a large factor. This issue will be discussed in Sects. 7 and 8.

3 The planetary rhythm of the Titius–Bode rule

Titius (1766) and later Bode (1772) noted that the semi-major axes $a_n$ of the planets of the solar system are function of the planetary sequence number $n$. Adding 4 to the series 0, 3, 6, 12, 24, 48, 96, 192 and 384 and dividing the result by 10 gives a series that approximately reproduces the semi-major axis length of the planets in astronomical units (1 AU = Sun–Earth average distance). The Titius–Bode rule for the orbital semi-major axis length, $a_n$, is a power-law equation that can be written as

$$a_n = 0.4 + 0.3 \times 2^n,$$

with $n = -\infty, 0, 1, 2, 3, 4, 5, 6, 7$, where $n = -\infty$ refers to Mercury, $n = 0$ to Venus, $n = 1$ to Earth, etc. As Table 1 shows, the Titius–Bode empirical rule successfully predicts the orbital semi-major axis length for all the planets and dwarf planets except for Neptune.

When the Titius–Bode rule was proposed (1766–1772) the dwarf planet Ceres (in the asteroid belt) and the Jovian planet Uranus were unknown. Indeed, the idea that undiscovered planets could exist between the orbits of Mars and Jupiter and beyond Saturn was strongly suggested by Bode in 1772. The curious gap separating Mars and Jupiter had, however, already been noted by Kepler.

The astronomers looked for new planets taking into account the predictions of the Titius–Bode rule. In 1781

<table>
<thead>
<tr>
<th>Planet</th>
<th>$n$</th>
<th>Titius–Bode rule $a_n$ (AU)</th>
<th>Observations $a$ (AU)</th>
<th>Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>$-\infty$</td>
<td>0.40</td>
<td>0.387</td>
<td>(3.3 %)</td>
</tr>
<tr>
<td>Venus</td>
<td>0</td>
<td>0.70</td>
<td>0.723</td>
<td>(3.18 %)</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>(0 %)</td>
</tr>
<tr>
<td>Mars</td>
<td>2</td>
<td>1.60</td>
<td>1.524</td>
<td>(5.0 %)</td>
</tr>
<tr>
<td>Ceres</td>
<td>3</td>
<td>2.80</td>
<td>2.77</td>
<td>(1.1 %)</td>
</tr>
<tr>
<td>Jupiter</td>
<td>4</td>
<td>5.20</td>
<td>5.204</td>
<td>(0.1 %)</td>
</tr>
<tr>
<td>Saturn</td>
<td>5</td>
<td>10.00</td>
<td>9.582</td>
<td>(4.4 %)</td>
</tr>
<tr>
<td>Uranus</td>
<td>6</td>
<td>19.60</td>
<td>19.201</td>
<td>(2.1 %)</td>
</tr>
<tr>
<td>Neptune</td>
<td>?</td>
<td>?</td>
<td>30.047</td>
<td>?</td>
</tr>
<tr>
<td>Pluto</td>
<td>7</td>
<td>38.80</td>
<td>39.482</td>
<td>(1.7 %)</td>
</tr>
</tbody>
</table>
Herschel (Dreyer, 1912) discovered Uranus, and in 1801 Piazzi (1801) discovered the dwarf planet Ceres. Both Ceres and Uranus fit the predictions of the Titius–Bode rule relatively well.

In the early 19th century, following Herschel and Piazzi’s discoveries, the Titius–Bode rule became widely accepted as a “law” of nature. However, the discovery of Neptune in 1846 created a severe problem because its semi-major axis length $a_{Ne} = 30.047$ AU does not satisfy the Titius–Bode prediction for $n = 7$, $a_7 = 38.80$ AU. The discovery of Pluto in 1930 confounded the issue still further. In fact, Pluto’s semi-major axis length, $a_{pl} = 39.482$ AU, would be inconsistent with the Titius–Bode rule unless Pluto is given the position $n = 7$ that the rule had predicted for Neptune (see Table 1).

The Titius–Bode rule is clearly imperfect or incomplete and no rigorous theoretical explanation of it still exists. However, it is unlikely that the relationship among the planets of the solar system that it approximately models is purely coincidental. Very likely any stable planetary system may satisfy a Titius–Bode-type relationship due to a combination of orbital resonances and shortage of degrees of freedom. Dubrulle and Graner (1994a, b) have shown that Titius–Bode-type rules could be a consequence of collapsing-cloud models of planetary systems possessing two symmetries: rotational invariance and scale invariance.

### 4 The asteroid belt “mirror” symmetry rule

Following the discovery of Ceres in 1801, numerous asteroids were discovered at approximately the same orbital distance. The region in which these asteroids were found lies between Mars and Jupiter and it is known as the asteroid belt. No planet could form in this region because of the gravitational perturbations of Jupiter that has prevented the accretion of the asteroids into a small planet. Ceres, with its spherical shape of $\sim 500$ km radius, is the largest asteroid and the only dwarf planet in the inner solar system.

A curious mathematical relationship linking the four terrestrial inner planets (Mercury, Venus, Earth and Mars) and the four giant gaseous outer planets (Jupiter, Saturn, Uranus and Neptune) exists (Geddes and King-Hele, 1983). The semi-major axes of these eight planets appear to reflect about the asteroid belt. This mirror symmetry associates Mercury with Neptune, Venus with Uranus, Earth with Saturn and Mars with Jupiter. Geddes and King-Hele (1983) found that the mutual relations among the planets could all be approximately given as relations between the mean frequency notes in an octave: $b = 2\exp(1/8)$.

For example, using the semi-major axis lengths reported in Table 1 for the eight planets and labeling these distances with the first two letters of the planet’s name, it is easy to obtain

$$
\begin{align*}
Me \times Ne &= 1.214 \cdot Ea \times Sa \\
Ve \times Ur &= 1.194 \cdot Me \times Ne \\
Ea \times Sa &= 1.208 \cdot Ma \times Ju,
\end{align*}
$$

where we have $b^2 \approx 1.19$, and

$$
\begin{align*}
Ve \times Ma &= 2.847 \cdot Me \times Ea \\
Sa \times Ne &= 2.881 \cdot Ju \times Ur,
\end{align*}
$$

where we have $b^{12} \approx 2.83$. Combining the equations yields

$$
\begin{align*}
Me \times Ne &\approx Ve \times Ur \cdot Ea \times Sa \\
Ea \times Sa &\approx Me \times Ne \div Ma \times Ju
\end{align*}
$$

and

$$
\begin{align*}
Me \times Ea &\approx Ju \times Ur \\
Ve \times Ma &\approx Sa \times Ne.
\end{align*}
$$

These relations relate the four inner and the four outer planets of the solar system. Even if the Geddes and King-Hele rule is not perfect, it does suggest the existence of a specific ordered structure in the planetary system where the asteroid belt region acts as a kind of mirroring boundary condition between the inner and outer regions of the solar system.

Geddes and King-Hele (1983) concluded that “the significance of the many near-equalities is very difficult to assess. The hard-boiled may dismiss them as mere playing with numbers; but those with eyes to see and ears to hear may find traces of something far more deeply interwoven in the fact that the average interval between the musical notes emerges as the only numerical constant required – a result that would surely have pleased Kepler.”

### 5 The matrix of planetary resonances

Molchanov (1968, 1969a) showed that the periods of the planets could be approximately predicted with a set of simple linear equations based on integer coefficients describing the mutual planetary resonances. Molchanov’s system is reported below:

$$
\begin{pmatrix}
1 & -1 & -2 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -3 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 & -6 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & 2 & -5 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -7 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -3
\end{pmatrix}
\begin{pmatrix}
\omega_{Me} \\
\omega_{Ve} \\
\omega_{Ea} \\
\omega_{Sa} \\
\omega_{Ju} \\
\omega_{Ma} \\
\omega_{Er} \\
\omega_{pl}
\end{pmatrix} =
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix},
$$

where $\omega = T^{-1}$ is the orbital frequency corresponding to the planetary period $T$. By imposing $\omega_{Ea}^{-1} = T_{Ea} = 1$ yr the system
(Eq. 6) predicts the following orbital periods:

<table>
<thead>
<tr>
<th>period</th>
<th>calculated</th>
<th>observed</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{Me}$</td>
<td>2484/10332</td>
<td>0.240</td>
<td>0.241</td>
</tr>
<tr>
<td>$T_{Ve}$</td>
<td>2484/4044</td>
<td>0.614</td>
<td>0.615</td>
</tr>
<tr>
<td>$T_{Es}$</td>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$T_{Ma}$</td>
<td>2484/1320</td>
<td>1.880</td>
<td>1.880</td>
</tr>
<tr>
<td>$T_{Na}$</td>
<td>2484/210</td>
<td>11.83</td>
<td>11.86</td>
</tr>
<tr>
<td>$T_{Sa}$</td>
<td>2484/84</td>
<td>29.57</td>
<td>29.46</td>
</tr>
<tr>
<td>$T_{Us}$</td>
<td>2484/30</td>
<td>82.80</td>
<td>84.01</td>
</tr>
<tr>
<td>$T_{Ne}$</td>
<td>2484/15</td>
<td>165.6</td>
<td>164.8</td>
</tr>
<tr>
<td>$T_{Pi}$</td>
<td>2484/10</td>
<td>248.4</td>
<td>248.1</td>
</tr>
</tbody>
</table>

where the last column gives the observed orbital periods of the planets in years. The absolute percent divergence between the predicted and observed orbital periods is given in parentheses.

Using simple linear algebra, the system (Eq. 6) can also be used to find alternative resonance relations. For example, summing the first two rows gives the following relation between Mercury, Earth, Mars and Jupiter: $\omega_{Me} - 2\omega_{Es} - 4\omega_{Ma} - \omega_{Sa} = 0$.

Molchanov (1968) showed that analogous tables of integers work also for describing planetary satellite systems such as the moon systems of Jupiter and Saturn. The provided physical explanation was that the resonant structure in a gravitationally interacting oscillating system could be inevitable under the action of dissipative perturbations of mutually comparable size. However, Molchanov (1969a) noted that alternative resonance relations yielding slightly different results could also be formulated. Nevertheless, even if it is the case that the system (Eq. 6) is neither unique nor perfectly descriptive of the orbital characteristics of the planets of the solar system, it does suggest that the planets are mutually synchronized. Molchanov (1969b) quantitatively evaluated that the probability of formation of a given resonant structure by chance is not very likely: the probability that the resonant structure of the solar system could emerge as a random chance was calculated to be less than $p = 10^{-10}$.

6 The gravitational harmonics of the solar system

The simplest way to determine whether the solar system is characterized by a harmonic order is to study its natural frequencies and find out whether they obey some general rule. The main set of frequencies that characterize the solar planetary system can be found by studying the power spectra of physical measures that are comprehensive of the motion of all planets such as the functions describing the dynamics of the Sun relative to the center of mass of the solar system. In fact, the Sun is wobbling around the center of mass of the solar system following a very complex trajectory due to the gravitational attraction of all planets. Figure 3 shows the wobbling of the Sun during specific periods.

Several functions of the coordinates of the Sun relative to the center of mass of the solar system can be chosen such as the distance, the speed, the angular momentum, etc. (e.g., Jose, 1965; Bucha et al., 1985). However, simple mathematical theorems establish that generic functions of the orbits of the planets must by necessity share a common set of planetary frequencies. Only the amplitudes associated with each harmonic are expected to depend on the specific chosen observable. Thus, unless one is interested in a given observable for a specific purpose, any general function of the orbits of the planets should suffice to determine the main harmonic set describing the planetary motion of the solar system as a whole.

Herein I extend the frequency analysis of the Sun’s motion made in Bucha et al. (1985) and Scafetta (2010). The JPL’s HORIZONS Ephemeris system is used to calculate the speed of the Sun relative to the center of mass of the solar system from 12 December 8002 BC to 24 April 9001 AD (100-day steps). Power spectra are evaluated using the periodogram and the maximum entropy method (Press et al., 1997).

Figure 4a depicts the result and highlights the main planetary frequencies of the solar system. Slightly different values may be found using different observables and subintervals of the analyzed period because of statistical variability and because of the relative amplitude of the frequencies’ change with the specific function of the planets’ orbits that are chosen for the analysis. An estimate of the statistical theoretical error associated with each measured frequency could be obtained using the Nyquist theorem of the Fourier analysis and it is given by $\nabla f = \pm 1/2L$, where $L = 17003$ yr is the length of the analyzed time sequence. Thus, if $P_0$ is the central estimate of a period, its range is given by $P \approx P_0 \pm P_0^2/2L$ (cf. Tan and Cheng, 2012).

Several spectral peaks can be recognized, such as the $\sim 1.092$ yr period of the Earth–Jupiter conjunctions; the $\sim 9.93$ and $\sim 19.86$ yr periods of the Jupiter–Saturn spring (half synodic) and synodic cycles, respectively; the $\sim 11.86$, $\sim 29.5$, $\sim 84$ and $\sim 165$ yr orbital period of Jupiter, Saturn, Uranus and Neptune, respectively; the $\sim 61$ yr cycle of the tidal beat between Jupiter and Saturn; and the periods corresponding to the synodic cycle between Jupiter and Neptune ($\sim 12.8$ yr), Jupiter and Uranus ($\sim 13.8$ yr), Saturn and Neptune ($\sim 35.8$ yr), Saturn and Uranus ($\sim 45.3$), and Uranus and Neptune ($\sim 171.4$ yr), as well as many other cycles including the spring (half-synodic) periods. Additional spectra peaks at $\sim 200–220$, $\sim 571$, $\sim 928$ and $\sim 4200$ yr are also observed. Clustered frequencies are typically observed. For example, the ranges $42–48$ yr, $54–70$ yr, $82–100$ yr (Gleissberg cycle) and $150–230$ yr (Suess–de Vries cycle) are clearly observed in Fig. 4 and are also found among typical main solar activity and aurora cycle frequencies (Ogurtsov et al., 2002; Scafetta and Willson, 2013a). The subannual planetary harmonics together with their spectral coherence with satellite total solar irradiance records and other solar records are discussed in Scafetta and Willson (2013b, c), and are not reported here.

The curious fact is that the numerous spectral peaks observed in the solar motion do not seem to be randomly distributed. They could be approximately reproduced using a
A simple empirical harmonic formula of the type (Jakubcová and Pick, 1986)

\[ p_i = \frac{178.38}{i} \text{ yr}, \quad i = 1, 2, 3, \ldots, \tag{8} \]

where the basic period of \( \sim 178.38 \text{ yr} \) is approximately the period that Jose (1965) found in the Sun’s motion and in the sunspot record (cf. Charvátová and Hejda, 2014). A comparison between the observed frequencies and the prediction of the resonance model, Eq. (8), is shown in Fig. 4b.

Although Eq. (8) is not perfect, and not all the modeled frequencies are clearly observed in Fig. 4a, the good agreement observed between most of the observed periods and the harmonic model predictions suggests that the solar system is characterized by a complex synchronized harmonic structure. Jakubcová and Pick (1986) also noted that several spectral peaks in the solar motion approximately correspond to the periods of various solar and terrestrial phenomena suggesting that the Sun itself, and the Earth’s climate, could be modulated by the same planetary harmonics (see also Charvátová and Hejda, 2014). This issue is further discussed below.

7 The planetary synchronization and modulation of the \( \sim 11 \text{ yr} \) solar cycle

In the 19th century, solar scientists discovered that sunspot activity is modulated by a quasi-11 yr oscillation called the Schwabe cycle. In a letter to Mr. Carrington, Wolf (1859) proposed that the observed solar oscillation could be caused by the combined influence of Venus, Earth, Jupiter and Saturn upon the Sun.

The planetary theory of solar variation is today not favored among solar scientists because, according to Newtonian
Figure 4. (A) Periodogram (red) and the maximum entropy method (blue) of the speed of the Sun relative to the center of mass of the solar system from Dec 12 8002 BC to 24 Apr 9001 AD. For periods larger than 200 yr the periodogram becomes unstable and is thus not shown. (B) Comparison between the frequencies observed and listed in (A) in the range 3 to 200 yr (red) and the frequency predictions of the resonance model Eq. (8) (blue). Note the good spectral coherence of the harmonic model with the observed frequencies.

physics, the planets appear too far from the Sun to modulate its activity, for example by gravitationally forcing the Sun’s tachocline (Callebaut et al., 2012). The planets could modulate solar activity only if a mechanism exists that strongly amplifies their gravitational and/or electromagnetic influence on the Sun. Scafetta (2012c) showed that a strong amplification mechanism could be derived from the mass–luminosity relation: the gravitational energy dissipated by planetary tides on the Sun was proposed to modulate the nuclear fusion rate yielding a variable solar luminosity production. It was calculated that the proposed mechanism could yield a $4 \times 10^6$ energetic amplification of the tidal signal. The derived oscillating luminosity signal could be sufficiently strong to modulate the Sun’s tachocline and convective zone (cf. Abreu et al., 2012; Mörner, 2013; Solheim, 2013a). Electromagnetic interactions between the planets and the Sun via Parker’s spiral magnetic field of the heliosphere, which could be modulated by functions related to the wobbling dynamics of the Sun such as its speed, jerk, etc., could also be possible in principle. Evidence for planet-induced stellar activity has been also observed in other stars (e.g., Scharf, 2010; Shkolnik et al., 2003, 2005).

It is important to stress that the contemporary view of solar science is that solar magnetic and radiant variability is
intrinsically chaotic, driven by internal solar dynamics alone and characterized by hydromagnetic solar dynamo models (Tobias, 2002). However, as also admitted by solar physicists (e.g., de Jager and Versteegh, 2005; Callebaut et al., 2012), present hydromagnetic solar dynamo models, although able to generically describe the periodicities and the polarity reversal of solar activity, are not yet able to quantitatively explain the observed solar oscillations. For example, they do not explain why the Sun should present an ~11 yr sunspot cycle and a ~22 yr Hale solar magnetic cycle. Solar dynamo models are able to reproduce a ~11 yr oscillation only by choosing specific values for their free parameters (Jiang et al., 2007). These dynamo models are not able to explain also the other solar oscillations observed at multiple scales such as the 50–140 yr Gleissberg cycle, the 160–260 yr Suess–de Vries cycle, the millennial solar cycles, etc. (cf. Ogurtsov et al., 2002), nor are they able to explain the phases of these cycles. Thus, the present solar dynamo theories appear to be incomplete. They cannot predict solar activity and they have not been able to explain the complex variability of the solar dynamo including the emergence of the ~11 yr oscillation. Some mechanism, which is still missed in the solar dynamo models, is needed to inform the Sun that it needs to oscillate at the observed specific frequencies and at the observed specific phases.

However, since Wolf (1859), several studies have highlighted that the complex variability of the solar dynamo appears to be approximately synchronized to planetary harmonics at multiple timescales spanning from a few days to millennia (e.g., Abreu et al., 2012; Bigg, 1967; Brown, 1900; Charvátová, 2009; Charvátová and Hejda, 2014; Fairbridge and Shirley, 1987; Hung, 2007; Jakubcová and Pick, 1986; Jose, 1965; Scafetta, 2010, 2012a, b, c, d, 2013b; Salvador, 2013; Scafetta and Willson, 2013b, a, c; Sharp, 2013; Solheim, 2013a; Tan and Cheng, 2012; Wilson, 2013a; Wolff and Patrone, 2010; and others).

Hung (2007) also reported that 25 of the 38 largest known solar flares were observed to start when one or more tide-producing planets (Mercury, Venus, Earth, and Jupiter) were either nearly above the event positions (less than 10 deg. longitude) or at the opposing side of the Sun. Hung (2007) estimated that the probability for this to happen at random was 0.039 % and concluded that “the force or momentum balance (between the solar atmospheric pressure, the gravity field, and magnetic field) on plasma in the looping magnetic field lines in solar corona could be disturbed by tides, resulting in magnetic field reconnection, solar flares, and solar storms.”

As Wolf (1859) proposed, the ~11 yr solar cycle could be produced by a combined influence of Venus, Earth, Jupiter and Saturn. There are two main motivations for this proposal:

1. The first model relating the 11 yr solar cycle to the configuration of Venus, Earth and Jupiter was proposed by Bendandi (1931); later Bollinger (1952), Hung (2007) and others developed equivalent models. It was observed that Venus, Earth and Jupiter are the three major tidal planets (e.g., Scafetta, 2012c). By taking into account the combined alignment of Venus, Earth and Jupiter, it is easy to demonstrate that the gravitational configuration of the three planets repeats every
Figure 6. The three-spectral-peak structure of Schwabe’s ~11 yr sunspot cycle as resolved by power spectra estimated using the maximum entropy method (MEM) and the periodogram (Press et al., 1997). The two side peaks at ~9.93 yr and ~11.86 yr correspond to the periods of Jupiter’s and Saturn’s spring tide and of Jupiter’s orbital tide on the Sun, respectively (cf. Scafetta, 2012b, c; Solheim, 2013a). Daily, monthly and yearly resolved sunspot number records are used covering periods from 1700 to 2013: http://sidc.oma.be/sunspot-data/.

\[ P_{\text{VEJ}} = \left( \frac{3}{P_{\text{Ve}}} - \frac{5}{P_{\text{Ea}}} + \frac{2}{P_{\text{Ju}}} \right)^{-1} = 22.14 \text{ yr}, \]  

where \( P_{\text{Ve}} = 224.701 \text{ days} \), \( P_{\text{Ea}} = 365.256 \text{ days} \) and \( P_{\text{Ju}} = 4332.589 \text{ days} \) are the sidereal orbital periods of Venus, Earth and Jupiter, respectively (Scafetta, 2012c). The 22.14 yr period is very close to the ~22 yr Hale solar magnetic cycle. Moreover, because the configurations Ea–Ve–Sun–Ju and Sun–Ve–Ea–Ju are equivalent about the tidal potential, the tidal cycle presents a recurrence of half of the above value (i.e., a period of 11.07 yr). This is the average solar cycle length observed since 1750 (e.g., Scafetta, 2012b). Figure 5 shows that a measure based on the most aligned days among Venus, Earth and Jupiter is well correlated, in phase and frequency, with the ~11 yr sunspot cycle: for details about the Venus–Earth–Jupiter 11.07 yr cycle see Battistini (2011, Bemdandi’s model), Bollinger (1952), Hung (2007), Scafetta (2012c), Salvador (2013), Wilson (2013a) and Tattersall (2013).

2. The main tides generated by Jupiter and Saturn on the Sun are characterized by two beating oscillations: the tidal oscillation associated with the orbital period of Jupiter (~11.86 yr period) and the spring tidal oscillation of Jupiter and Saturn (~9.93 yr period) (Brown, 1900; Scafetta, 2012c). Scafetta (2012b, c) used detailed spectral analysis of the sunspot monthly record since 1749 and showed that the ~11 yr solar cycle is constrained by the presence of two spectral peaks close to the two theoretical tidal periods deduced from the orbits of Jupiter and Saturn (see Fig. 6). These two frequencies modulate the main central cycle at ~10.87 yr period. The beat generated by the superposition of the three harmonics is characterized by four frequencies at about 61, 115, 130, and 983 yr periods that are typically observed in solar records (e.g., Ogurtsov et al., 2002; Scafetta, 2012b). Scafetta (2012b) proposed a harmonic model for solar variability based on three frequencies at periods of ~9.93, ~10.87 and ~11.86 yr. The phases of the three harmonics were determined from the conjunction date of Jupiter and Saturn (2000.475), the sunspot record from 1749 to 2010 (2002.364) and the perihelion date of Jupiter (1999.381), respectively. This simple three-frequency solar model not only oscillates with a ~11 yr cycle, as it should by mathematical construction, but it also manifests a complex multidecadal to millennial beat modulation that has been shown to hindcast all major patterns observed in both solar and climate records throughout the Holocene (Scafetta, 2012b). For example, the model was shown to efficiently hindcast: (1) the quasi-millennial oscillation (~983 yr) found in both climate and solar records (Bond et al., 2001); (2) the grand solar minima during the last millennium such as the Oort, Wolf, Spörer, Maunder and Dalton minima; (3) seventeen ~115 yr long oscillations found
in a detailed temperature reconstruction of the Northern Hemisphere covering the last 2000 yr; and (4) the \( \sim 59–63 \) yr oscillation observed in the temperature record since 1850 and other features. Scafetta’s (2012b) three-frequency solar model forecasts that the Sun will experience another moderate grand minimum during the following decades and will return to a grand maximum in the 2060s similar to the grand maximum experienced in the 2000s (see Fig. 7b).

Solheim (2013a) observed that if the longer sunspot yearly resolved record is used (1700–2012), then the central spectral peak observed in Fig. 6 at \( \sim 10.87 \) yr could be split into two peaks as \( \sim 11.01 \) yr and \( \sim 10.66 \) yr period. My own re-analysis of the periodogram of the sunspot annual record

Figure 7. Scafetta (2012b) three-frequency solar model (red). (A) Against the Northern Hemisphere temperature reconstruction by Ljungqvist (2010) (black). The bottom section depicts a filtering of the temperature reconstruction (black) that highlights the 115 yr oscillation (blue). (B) The same solar model (red) is plotted against the HadCRUT4 global surface temperature (black) merged in 1850–1900 with the proxy temperature model by Moberg et al. (2005) (blue). The green curves highlight the quasi-millennial oscillation with its skewness that approximately reproduces the millennial temperature oscillation from 1700 to 2013. Note the hindcast of the Maunder and Dalton solar minima and relative cool periods as well as the projected quasi 61 yr oscillation from 1850 to 2150. Adapted from Scafetta (2013a, b).
since 1700 shows that the split produces a secondary peak at $10.52 \pm 0.2\, \text{yr}$ and a main peak at $11.00 \pm 0.2\, \text{yr}$. This result suggests that the central peak at $\sim 10.87\, \text{yr}$, which was interpreted in Scafetta (2012b, c) as being produced by an internal dynamo cycle, could indeed emerge from the Venus–Earth–Jupiter recurrent cycles at $\sim 11.07\, \text{yr}$ period plus a possible minor cycle at $\sim 10.57\, \text{yr}$ period. Figure 4 shows that these two spectral peaks, plus another one at $\sim 11.26\, \text{yr}$ period, are among the planetary harmonics. This issue needs further analysis. As for the ocean tidal system on Earth, it is possible that multiple planetary oscillations regulate the $\sim 11\, \text{yr}$ solar cycle.

The physical meaning of the three-frequency solar model is that solar variability at the multidecadal to millennial scales is mostly determined by the interference among the harmonic constituents that make up the main $\sim 11\, \text{yr}$ solar oscillation. When these harmonics interfere destructively the Sun enters into a prolonged grand minimum; when they interfere constructively the Sun experiences a grand maximum. Additional oscillations at $\sim 45$, $\sim 85$, $\sim 170$ and $\sim 210\, \text{yr}$ period, also driven by the other two giant planets, Uranus and Neptune (see Fig. 4), have been observed in long solar and auroral records (Ogurtsov et al., 2002; Scafetta, 2012b; Scafetta and Willson, 2013a) but not yet included to optimize the three-frequency solar model.

Note that the three-frequency solar model proposed by Scafetta (2012b) is a semi-empirical model because it is based on the two main physical tidal harmonics generated by Jupiter and Saturn plus a statistically estimated central $\sim 11\, \text{yr}$ solar harmonic. Therefore, this model is based on both astronomical and empirical considerations, and its hindcasting capability have been tested for both centuries and millennia. Alternative empirical models of solar variability directly based on long-range harmonics determined using power spectra and linear regressions of solar records have been also proposed (e.g., Scafetta and Willson, 2013a; Solheim, 2013a; Salvador, 2013; Steinhilber and Beer, 2013). However, models based on as many astronomical and physical considerations as possible should be preferred to purely statistical or regressive models because the former are characterized by a lower number of degrees of freedom than the latter for the same number of modeled harmonics.

The proposed semi-empirical and empirical harmonic solar models agree about the fact that the Sun is entering into a period of grand minimum. Indeed, the latest sunspot cycles #19–24 are closely correlated to the sunspot cycles #1–5 immediately preceding the Dalton Minimum (1790–1830) (see Fig. 8). Battistini (2011) noted that the 11 yr solar cycle model proposed by Bendandi (1931) based on the Venus–Earth–Jupiter configuration is slightly out of phase with both the sunspot cycles #2–4 preceding the Dalton Minimum and with the sunspot cycles #22–24. This result may also be further evidence suggesting that the situation preceding the

Figure 8. Comparison between latest sunspot cycles #19–24 (black) and the sunspot cycles #1–5 (red) immediately preceding the Dalton Minimum (1790–1830). A new Dalton-like solar minimum is likely approaching and may last until 2045. The 211 yr temporal lag approximately corresponds to a Suess–de Vries solar cycle, which approximately corresponds to the $\sim 210\, \text{yr}$ beat period between the $\sim 60\, \text{yr}$ Jupiter–Saturn beat (Figs. 2c and 4a) and the 84 yr Uranus orbital cycle. From Scafetta (2012b).
Dalton Minimum is repeating today and could be anticipated by a planetary configuration.

8 Astronomically based semi-empirical harmonic climate models

As already understood since antiquity (cf. Ptolemy, 2nd century), Kepler (1601) recognized that the moon plays a crucial role in determining the ocean tidal oscillations, and in doing so, he anticipated Newton (1687) in conceiving invisible forces (gravity and electromagnetism) that could act at great distances. Kepler also argued that the climate system could be partially synchronized to multiple planetary harmonics (Kepler, 1601, 1606). The main long-scale harmonics that Kepler identified were a ~20 yr oscillation, a ~60 yr oscillation and a quasi-millennial oscillation. These oscillations were suggested by the conjunctions of Jupiter and Saturn and by historical chronological considerations (Kepler, 1606; Ma’Sar, 9th century). The quasi-millennial oscillation was associated with the slow rotation of the trigon of the conjunctions of Jupiter and Saturn, and Kepler (1606) claimed that this cycle was ~800 yr long (see Fig. 2c). Kepler’s calculations were based on the tropical orbital periods of Jupiter and Saturn, which is how the orbits of Jupiter and Saturn are seen from the Earth. However, using the sidereal orbital periods this oscillation should be 850–1000 yr long (Scafetta, 2012a), as suggested in the power spectrum analysis shown in Fig. 4. Since antiquity equivalent climatic oscillations have been noted (Iyengar, 2009; Ma’Sar, 9th century; Temple, 1998) and inserted in traditional calendars. For example, the Indian and Chinese traditional calendars are based on a 60 yr cycle known in the Indian tradition as the Brihaspati (which means Jupiter cycle).

The existence of climatic oscillations at about 10, 20, 60 and 1000 yr (and others) have been confirmed by numerous modern studies analyzing various instrumental and proxy climatic records such as the global surface temperature, the Atlantic Multidecadal Oscillation (AMO), the Pacific Decadal Oscillation (PDO), the North Atlantic Oscillation (NAO), ice core records, tree ring records, sea level records, fishery records, etc. (e.g., Bond et al., 2001; Chylek et al., 2011; Klyashtorin et al., 2009; Knudsen, 2011; Jevrejeva et al., 2008; Mörner, 1989; Scafetta, 2012a, 2013c; Wyatt and Curry, 2013). Indeed, numerous authors have also noted a correlation at multiple scales between climate oscillations and planetary functions – for example, those related to the dynamics of the Sun relative to the barycenter of the solar system (e.g., Charvátová, 1997; Charvátová and Hejda, 2014; Fairbridge and Shirley, 1987; Jakubcová and Pick, 1986; Landscheidt, 1989; Scafetta, 2010, 2012b; Solheim, 2013a).

In particular, global surface temperature records, which are available from 1850, present at least four major spectral peaks at periods of about 9.1, 10–11, 20 and 60 yr, plus three minor peaks at about 12, 15 and 30 yr (see Fig. 1 in Scafetta, 2013b, which is partially reproduced in Solheim, 2013b). Subdecadal astronomical oscillations are also observed in climatic records (Scafetta, 2010). In addition, multisecular and millennial oscillations (e.g., there are major ~115 and ~983 yr oscillations and others) can be deduced from paleoclimatic proxy temperature models. As also shown in Fig. 4, these oscillations can be associated with planetary harmonics (Scafetta, 2010, 2012b). Astronomically based semi-empirical harmonic models to reconstruct and forecast climatic changes are being proposed by several authors (e.g., Abdusamatov, 2013; Akasofu, 2013; Lüdecke et al., 2013; Salvador, 2013; Scafetta, 2010, 2012a, b, d, 2013a; Solheim, 2013a).

For example, Scafetta (2013b) proposed a semi-empirical harmonic climate model based on astronomical oscillations plus an anthropogenic and volcano contribution. In its latest form this model is made of the following six astronomically deduced harmonics with periods of 9.1, 10.4, 20, 60, 115, 983 yr:

\[
\begin{align*}
  h_{01}(t) & = 0.044 \cos(2\pi(t - 1997.82)/9.1) \\
  h_{10,4}(t) & = 0.030 \cos(2\pi(t - 2002.93)/10.4) \\
  h_{20}(t) & = 0.043 \cos(2\pi(t - 2001.43)/20) \\
  h_{60}(t) & = 0.111 \cos(2\pi(t - 2001.29)/60) \\
  h_{115}(t) & = 0.050 \cos(2\pi(t - 1980)/115) \\
  h_{983}(t) & = 0.350 \cos(2\pi(t - 2060)/760).
\end{align*}
\]

In the last equation a 760 yr period from 1680 to 2060 is used instead of a 983 yr period because the millennial temperature oscillation is skewed. While its maximum is predicted to occur in 2060, the minimum occurs around 1680 during the Maunder Minimum (1645–1715) (see Fig. 7a above and Fig. 8 in Humlum et al., 2011).

The 9.1 yr cycle was associated with a soli-lunar tidal oscillation (e.g., Scafetta, 2010, 2012d). The rationale was that the lunar nodes complete a revolution in 18.6 yr and the Saros soli-lunar eclipse cycle completes a revolution in 18 yr and 11 days. These two cycles induce 9.3 yr and 9.015 yr tidal oscillations corresponding respectively to the Sun–Earth–Moon and Sun–Moon–Earth symmetric tidal configurations. Moreover, the lunar apsidal precession completes one rotation in 8.85 yr, causing a corresponding lunar tidal cycle. The three cycles cluster between 8.85 and 9.3 yr periods producing an average period around 9.06 yr. This soli-lunar tidal cycle peaked in 1997–1998, when the solar and lunar eclipses occurred close to the equinoxes and the tidal torque was stronger because centred on the Equator. Indeed, the ~9.1 yr temperature cycle was found to peak in 1997.82, as expected from the soli-lunar cycle model (Scafetta, 2012d).

The other five oscillations of Eq. (10) were deduced from solar and planetary oscillations. The 10.4 yr cycle appears to be a combination of the ~10 yr Jupiter–Saturn spring cycle and the ~11 yr solar cycle and peaks in 2002.93 (i.e., ~1 yr after the maximum of solar cycle 23) that occurred in ~2002. The ~20 and ~60 yr temperature cycles are synchronized
with the ~20 and ~60 yr oscillations of the speed of the Sun relative to the center of mass of the solar system (Scafetta, 2010) and the ~61 yr beat cycle of the Jupiter–Saturn tidal function, which peaked around the 1880s, 1940s and 2000s (Scafetta, 2012b, c) (see also Fig. 7b). I note, however, that Wilson (2013b) proposed a complementary explanation of the ~60 yr climatic oscillation, which would be caused by planetary induced solar activity oscillations resonating with tidal oscillations associated to specific lunar orbital variations synchronized with the motion of the Jovian planets. The ~115 and ~983 yr oscillations are synchronized with both the secular and millennial oscillations found in climatic and solar proxy records during the Holocene (Scafetta, 2012b). The amplitude of the millennial cycle is determined using modern paleoclimatic temperature reconstructions (Ljungqvist, 2010; Moberg et al., 2005). The six oscillations of Eq. (10) are quite synchronous to the correspondent astronomical oscillations (see Fig. 7 and Scafetta, 2010, 2013b). Only the amplitudes of the oscillations are fully free parameters that are determined by regression against the temperature record. See Scafetta (2010, 2012b, 2013b) for details.

To complete the semi-empirical model, a contribution from anthropogenic and volcano forcings was added. It could be estimated using the outputs of typical general circulation models (GCMs) of the Coupled Model Intercomparison Project 5 (CMIP5) simulations, $m(t)$, attenuated by half, $\beta = 0.5$ (Scafetta, 2013b). The attenuation was required to compensate for the fact that the CMIP3 and CMIP5 GCMs do not reproduce the observed natural climatic oscillations (e.g., Scafetta, 2010, 2012b, 2013b). This operation was also justified on the ground that the CMIP5 GCMs predict an almost negligible solar effect on climate change and their simulations essentially model anthropogenic plus volcano radiative effects alone. Finally, the adoption of $\beta = 0.5$ was also justified by the fact that numerous recent studies (e.g., Chylek et al., 2011; Chylek and Lohmann, 2008; Lewis, 2013; Lindzen and Choi, 2011; Ring et al., 2012; Scafetta, 2013b; Singer, 2011; Spencer and Braswel, 2011; Zhou and Tung, 2012) have suggested that the true climate sensitivity to radiative forcing could be about half (~0.7–2.3 °C for CO2 doubling) of the current GCM estimated range (~1.5 to 4.5 °C; IPCC, 2013).

Scafetta’s (2013b) semi-empirical climate model was calculated using the following formula:

$$H(t) = h_{9.1}(t) + h_{10.4}(t) + h_{20}(t) + h_{60}(t) + h_{115}(t) + h_{983}(t) + \beta \times m(t) + \text{const.}$$

(11)

Figure 9 shows that the model (Eq. 11, red curve) successfully reproduces all of the decadal and multidecadal oscillating patterns observed in the temperature record since 1850, including the upward trend and the temperature standstill since 2000. However, the decadal and multidecadal temperature oscillations and the temperature standstill since ~2000

---

**Figure 9.** The semi-empirical model (Eq. 11) using $\beta = 0.5$ (red) attenuation of the CMIP5 GCM ensemble mean simulation vs. HadCRUT4 GST record from Jan 1860 to Nov 2013 (black). The cyan curve represents the natural harmonic component alone (Eq. 10). The green curve represents the CMIP5 GCM average simulation used by the IPCC in 2013. The model reconstructs the 20th century warming and all decadal and multidecadal temperature patterns observed since 1860 significantly better than the GCM simulations such as the standstill since ~1997, which is highlighted in the insert (cf. Scafetta, 2010, 2012d, 2013b).
are macroscopically missed by the CMIP5 GCM simulations adopted by the Intergovernmental Panel on Climate Change (IPCC, 2013) (cf. Scafetta, 2013b). As Fig. 9 shows, Eq. (11) projects a significantly lower warming during the 21st century than the CMIP5 average projection.

Alternative empirical models for the global surface temperature have been proposed by Scafetta (2010, 2012a, d, 2013a), Solheim (2013b), Akasofu (2013), Abdusamatov (2013), Lüdecke et al. (2013), Vahrenholt and Lüning (2013) and others. These models are based on the common assumption that the climate is characterized by specific quasi-harmonic oscillations linked to astronomical–solar cycles. However, they differ from each other in important mathematical details and physical assumptions. These differences yield different performances and projections for the 21st century. For example, Scafetta’s (2010, 2012a, d, 2013a, b) models predict a temperature standstill until the 2030s and a moderate anthropogenic warming from 2000 to 2100 modulated by natural oscillations such as the ~ 60 yr cycle (see the red curve in Fig. 9). Scafetta’s model takes into account that the natural climatic variability, driven by a forecasted solar minimum similar to a moderate Dalton solar minimum or to the solar minimum observed during ~ 1910 (see Figs. 7b and 8) would yield a global cooling of ~ 0.4 °C from ~2000 to ~2030 (see cyan curve in Fig. 9), but this natural cooling would be mostly compensated by anthropogenic warming as projected throughout the 21st century by Scafetta’s β-attenuated model (see Eq. 11). Although with some differences, the climatic predictions of Solheim (2013b), Akasofu (2013) and Vahrenholt and Lüning (2013) look quite similar: they predict a steady to moderate global cooling from 2000 to 2030 and a moderate warming for 2100 modulated by a ~60 yr cycle. However, Abdusamatov (2013, Fig. 8) predicted an imminent cooling of the global temperature beginning from the year 2014 that will continue throughout the first half of the 21st century and would yield a Little Ice Age period from ~2050 to ~2110, when the temperature would be ~1.2 °C cooler than the 2000–2010 global temperature. Abdusamatov’s predicted strong cooling would be induced by an approaching Maunder-like solar minimum period that would occur during the second half of the 21st century. Steinhilber and Beer (2013) also predicted a grand solar minimum occurring during the second half of the 21st century, but it would be quite moderate and more similar to the solar minimum observed during ~ 1910; thus, this solar minimum will not be as deep as the Maunder solar minimum of the 17th century.

An analysis and comparison of the scientific merits of each proposed harmonic constituent solar and climate model based on astronomical oscillations elude the purpose of this paper and it is left to the study of the reader. In general, harmonic models based only on statistical, Fourier and regression analysis may be misleading if the harmonics are not physically or astronomically justified. Nonetheless, harmonic constituent models can work exceptionally well in reconstructing and forecasting the natural variability of a system if the dynamics of the system are sufficiently harmonic and the constituent physical/astronomical harmonics are identified with great precision. For example, the astronomically based harmonic constituent models currently used to predict the ocean tides are the most accurate predictive geophysical models currently available (Doodson, 1921; http://en.wikipedia.org/wiki/Theory_of_tides).

Scafetta (2012b, d, 2013b) carefully tested his solar and climate models based on astronomical oscillations using several hindcasting procedures. For example, the harmonic solar model was tested in its ability to hindcast the major solar patterns during the Holocene and the harmonic climate model was calibrated during the period 1850–1950 and its performance to obtain the correct 1950–2010 patterns was properly tested, and vice versa. Future observations will help to better identify and further develop the most reliable harmonic constituent climate model based on astronomical oscillations.

9 Conclusions

Pythagoras of Samos (Pliny the Elder, 77 AD) proposed that the Sun, the Moon and the planets all emit their own unique *hum* (orbital resonance; cf. Tattersall, 2013) based on their orbital revolution, and that the quality of life on Earth reflects somehow the tenor of the celestial *sounds* (from http://en.wikipedia.org/wiki/Musica_universalis). This ancient philosophical concept is known as *musica universalis* (universal music or music of the spheres). However, it is with Copernicus’ heliocentric revolution that the harmonic structure of the solar system became clearer. Kepler (1596, 1619) strongly advocated the *harmonices mundi* (the harmony of the world) concept from a scientific point of view.

Since the 17th century, scientists have tried to disclose the fundamental mathematical relationships that describe the solar system. Interesting resonances linking the planets together have been found. I have briefly discussed the Titius–Bode rule and other resonant relationships that have been proposed during the last centuries. In addition, planetary harmonics have been recently found in solar and climate records, and semi-empirical models to interpret and reconstruct the climatic oscillations, which are not modeled by current GCMs, have been proposed (e.g., Scafetta, 2013b).

How planetary harmonics could modulate the Sun and the climate on the Earth is still unknown. Some papers have noted that a tidal-torquing function acting upon hypothesized distortions in the Sun’s tachocline present planetary frequencies similar to those found in solar proxy and climate records (e.g., Abreu et al., 2012; Wilson, 2013a). However, whether planetary gravitational forces are energetically sufficiently strong to modulate the Sun’s activity in a measurable way remains a serious physical problem and reason for skepticism. Also, basic Newtonian physics, such as simple evaluations of tidal accelerations on just the Sun’s tachocline, does not
seem to support the theory due to the fact that planetary tidal accelerations on the Sun seem are too small (just noise) compared to the strengths of the typical convective accelerations (Callebaut et al., 2012).

However, the small gravitational perturbation that the Sun is experiencing are harmonic, and the Sun is a powerful generator of energy very sensitive to gravitational and electromagnetic variations. Thus, the Sun’s internal dynamics could synchronize to the frequency of the external forcings and it could work as a huge amplifier and resonator of the tenuous gravitational music generated by the periodic synchronized motion of the planets. Scafetta (2012c) proposed a physical amplification mechanism based on the mass–luminosity relation. In Scafetta’s model the Sun’s tachocline would be forced mostly by an oscillating luminosity signal emerging from the solar interior (cf. Wolff and Patrone, 2010). The amplitude of the luminosity anomaly signal driven by the planetary tides, generated in the Sun’s core and quickly propagating as acoustic-like waves in the radiative zone into the Sun’s tachocline, has to oscillate with the tidal and torquing planetary gravitational frequencies because function of the gravitational tidal potential energy dissipated in the solar interior. The energetic strength of this signal was estimated and found to be sufficiently strong to synchronize the dynamics of the Sun’s tachocline and, consequently, of the Sun’s convective zone. The quasi-harmonic and resonant structure observed in the solar system should further favor the emergence of collective synchronization patterns throughout the solar system and activate amplification mechanisms in the Sun and, consequently, in the Earth’s climate.

Although a comprehensive physical explanation has not been fully found yet, uninterrupted aurora records, solar records and long solar proxy records appear to be characterized by astronomical harmonics from monthly to the millennial timescales, and the same harmonics are also present in climate records, as has been found by numerous authors since the 19th century (e.g., Wolf, 1859; Brown, 1900; Abreu et al., 2012; Charvátová, 2009; Charvátová and Hejda, 2014; Fairbridge and Shirley, 1987; Hung, 2007; Jakubcová and Pick, 1986; Jose, 1965; Salvador, 2013; Scafetta, 2010, 2012a, b, c, d, 2013b; Scafetta and Willson, 2013b, a, c; Sharp, 2013; Solheim, 2013a; Tan and Cheng, 2012; Wilson, 2011, 2013a; Wolff and Patrone, 2010). Thus, gravitational and electromagnetic planetary forces should modulate both solar activity and, directly or indirectly, the electromagnetic properties of the heliosphere. The climate could respond both to solar luminosity oscillations and to the electromagnetic oscillations of the heliosphere and synchronize to them. The electromagnetic oscillations of the heliosphere and the interplanetary electric field could directly influence the Earth’s cloud system through a modulation of cosmic ray and solar wind, causing oscillations in the terrestrial albedo, which could be sufficiently large (about 1–3%) to cause the observed climatic oscillations (e.g., Mörner, 2013; Scafetta, 2012a, 2013b; Svensmark, 2007; Tinsley, 2008; Voiculescu et al., 2013).

Although the proposed rules and equations are not perfect yet, the results of this paper do support the idea that the solar system is highly organized in some form of complex resonant and synchronized structure. However, this state is dynamical and is continuously perturbed by chaotic variability, as it should be physically expected. Future research should investigate planets–Sun and space–climate coupling mechanisms in order to develop more advanced and efficient analytical and semi-empirical solar and climate models. A harmonic set made of the planetary harmonics listed in Fig. 4 plus the beat harmonics generated by the solar synchronization (e.g., Scafetta, 2012b) plus the harmonics deductible from the soli-lunar tides (e.g., Wang et al., 2012) perhaps constitutes the harmonic constituent group that is required for developing advanced astronomically based semi-empirical harmonic climate models.

As Pythagoras, Ptolemy, Kepler and many civilizations have conjectured since antiquity, solar and climate forecasts and projections based on astronomical oscillations appear physically possible. Advancing this scientific research could greatly benefit humanity.

Acknowledgements. The author thanks R. C. Willson (ACRIM science team) for support, the referees for useful suggestions and the editors for having organized the special issue Pattern in solar variability, their planetary origin and terrestrial impacts (Pattern Recognition in Physics, 2013; Editors: N.-A. Mörner, R. Tattersall, and J.-E. Solheim), where 10 authors try to further develop the ideas about the planetary–solar–terrestrial interaction.

Edited by: N.-A. Mörner
Reviewed by: two anonymous referees

References

Abdusamatov, Kh. I.: Grand minimum of the total solar irradiance leads to the little ice age, Geology & Geosciences, 28, 62–68, 2013.


Copernicus, N.: De revolutionibus orbium coelestium. (Johannes Petreius), 1543.


Kepler, J.: Mysterium Cosmographicum (The Cosmographic Mystery), 1596.


Kepler, J.: De Stella Nova in Pede Serpentarii (On the new star in Ophiuchus’s foot), 1606.


Newton, I.: Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy), 1687.


Piazzì, G.: Risultati delle Osservazioni della Nuova Stella (Palermo), 3–6, 1801.


