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# The sunspot cycle length – modulated by planets?

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**Abstract.** The Schwabe frequency band of the sunspot record since 1700 has an average period of 11.06 yr and contains four major cycles, with periods of 9.97, 10.66, 11.01 and 11.83 yr. Analysis of the O–C residuals of the timing of solar cycle minima reveals that the solar cycle length is modulated by a secular period of about 190 yr and the Gleissberg period of about 86 yr.

Based on a simple harmonic model with these periods, we predict that the solar cycle length will in average be longer during the 21st century. Cycle 24 may be about 12 yr long, while cycles 25 and 26 are estimated to be about 9 and 11 yr long. The following cycle is estimated to be 14 yr long. In all periods during the last 1000 yr, when the solar cycle length has increased due to the 190 yr cycle, a deep minimum of solar activity has occurred. This is expected to re-occur in the first part of this century.

The coherent modulation of the solar cycle length over a period of 400 yr is a strong argument for an external tidal forcing by the planets Venus, Earth, Jupiter and Saturn, as expressed in a spin-orbit coupling model.

# 1 Introduction

A possible relation between solar activity as manifested by sunspots and the Earth's climate has been discussed many times since William Herschel (1801) speculated on a possible connection. In recent times Reid (1987) showed, based on data on globally averaged sea surface temperature (SST), that the solar irradiance may have varied in phase with the 80–90 yr cycle represented by an envelope of the 11 yr solar activity cycle, called the Gleissberg cycle.

Friis-Christensen and Lassen (1991) investigated the relation between the sunspot numbers and Northern Hemisphere land temperature, and found similar variations, but with the temperature variations leading the sunspot numbers. They then discovered that using the solar cycle length (SCL) as an indicator of solar activity in the sense that a shorter cycle means higher activity, they could much better correlate with the NH land temperature variations. It was also demonstrated (Friis-Christensen and Lassen, 1991; Hoyt and Schatten, 1993; Larssen and Friis-Christensen, 1995) that the correlation between SCL and climate has probably been in operation for centuries. A statistical study of 69 tree ring sets, covering more than 594 yr, demonstrated that wider tree rings (better growth conditions) were associated with shorter sunspot cycles (Zhou and Butler, 1998).

In their study, Friis-Christensen and Lassen (1991) used a smoothed mean value for the SCL with the length of five solar cycles weighted 1-2-2-2-1. In a follow-up paper, Reichel et al. (2001) concluded that the right cause-and-effect ordering, in the sense of Granger causality, is present between the smoothed SCL and the cycle mean temperature anomaly for the Northern Hemisphere land air temperature in the 20th century at the 99 % significance level. This suggests that there may exist a physical mechanism linking solar activity to climate variations. However, at the turn of the century, a discrepancy between the SCL and NH land series developed (Thejll and Lassen, 2000; Thejll, 2009), because the short cycle 22 was followed by a much longer cycle 23, without sign of cooling.

Recognizing that averaged temperature series from different meteorological stations of variable quality and changing locations may contain errors and partially unknown phenomena derived from the averaging procedure, Butler (1994) proposed instead to use long series of high quality from single stations. He showed that this improved the correlation when used for temperature series for Armagh, which correlates strongly with the NH land temperature.

Archibald (2008) was the first to realize that the length of the previous sunspot cycle (PSCL) has a predictive power for the temperature in the next sunspot cycle for certain locations, if the raw (unsmoothed) value for the SCL is used. Based on the estimated longer SC23 than SC22, he predicted cooling during SC24 for some selected locations. A systematic study of the correlation for locations around the North Atlantic was published by Solheim et al. (2012). They found that maximum correlation was obtained with an 8–12 yr lag for locations around and in the North Atlantic, and found that a correlation with a lag of one solar cycle could explain 25 to 72 per cent of the temperature variance in that region. This one cycle lag could therefore be used for forecasting the temperature in the next solar cycle. Based on SC23 being considerably longer than SC22, they forecast a temperature decline during SC24 for the sites investigated.

In order to forecast the development of SCL for longer periods, it is necessary to investigate the long-term variability of the SCLs. This was done for the first time by Fairbridge and Hameed (1983), who found that the phase differences repeated after 16 sunspot cycles, or 178 yr, if they used minima as the start time for a cycle.

This was followed up by Richards et al. (2009), who used median trace analyses of the SCL and power spectrum analysis of the O–C residuals (as explained in Eq. 1). They found that the solar cycle length is controlled by periods of 188 and 87 yr. They concluded that the length of the solar cycle should increase gradually the next  $\approx 75$  yr. They did not discuss the origin of their determined periods.

Regarding the 11 yr sunspot period, many scientists have noticed the bimodal structure of the distribution of solar cycle length. According to analysis by Scafetta (2012), the sunspot length probability distribution consists of three periods of about 9.98, 10.9 and 11.86 yr. The side periods appear to be closely related to the spring period of Jupiter and Saturn, which has a range between 9.5 and 10.5 yr with a median length of 9.93 yr, and the sidereal period of Jupiter (about 11.86 yr). Scafetta (2012) proposed that the central cycle period is associated with a quasi 11 yr solar dynamo cycle, which is forced by the two cyclical side attractors with periods of 9.93 and 11.86 yr. He also suggested that the secular variations of the solar cycle amplitude and length are beat periods of the three solar cycle periods, and that it is possible to describe the secular variations of the sunspot cycle with these beat periods.

Scafetta's analysis covered the period 1755–2008 (solar cycles 1–23). In the following we will investigate the solar cycles for the longer period 1700–2010, and we will also investigate the O–C residuals all the way back to 1610 to search for period combinations or harmonics. Based on a simple harmonic model we will estimate the length of the next solar cycles. Finally we will discuss if the modulation of the SCL may be controlled by the planets, as proposed by Scafetta (2012) and Wilson et al. (2008).

#### 2 Data and methods

Yearly average sunspot numbers were downloaded from the Solar Influences Data Center (SIDC). The length and time of solar cycles were downloaded from http://www.ngdc.noaa.gov/stp/space-weather/solar-data/solar-indices/sunspot-numbers/cycle-data/table\_cycle-dates maximum-minimum.txt.

For the analysis of the sunspot number time series I have used the Period04 analysis package (Lenz and Breger, 2005), downloaded from the Period04 website at http://www.astro.univie.ac.at/dsn/Period04/. This program performs least square fitting of a number of frequencies, where initial frequencies may be determined by Fourier transform (FT) or given as input. Error analysis is done by an analytical formula (Breger et al., 1999) assuming an ideal case, or with a least square error calculation. The largest of the obtained errors is used.

The O–C technique for investigation of secular modulation of the SCL is described in detail in Richards et al. (2009). We follow their description and use the downloaded set of SCLs determined between the minima, and construct the O–C residuals cycle by cycle using the formula:

$$(O-C)_i = (t_i - t_0) - (N_i \times P_0), \tag{1}$$

where  $t_i$  is the end time of cycle no.  $N_i$ ,  $P_0$  is the reference period investigated, and  $C_i = t_0 + N_i \times P_0$ .

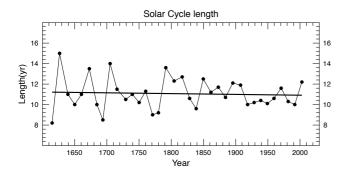
# 3 Results

## 3.1 The 11 yr cycle

The solar cycle length variation with time since 1610 is shown in Fig. 1. We notice large variations in the 17th and 18th centuries, but with a generally shorter length from about 1850. The data set covers a total of 36 cycles, and the mean length is  $11.06 \pm 1.5$  yr. In Fig. 2 we show the distribution of the SCL between solar minima. The median value is between 10.7 and 11.0 yr, but there are no observations in this range. This clearly indicates a double or multiple bell distribution.

The resulting periodogram of the sunspot numbers from 1700–2010 is shown in Fig. 3. We find, as did Scafetta (2012), a dominating band with periods 10-12 yr, where we identify four peaks:  $P_1 = 9.97 \pm 0.02$ ,  $P_2 = 10.66 \pm 0.02$ ,  $P_3 = 11.010 \pm 0.001$  and  $P_4 = 11.83 \pm 0.02$  yr. The errors are determined by an analytical formula (Breger et al., 1999). There is also a triplet of periods in an 8.5 yr band, and a triplet around 5.5 yr. The latter is most likely higher harmonics of three peaks in the 11 yr band.

The long period of  $53 \pm 0.6$  yr is best explained as a 4th subharmonics of  $P_2$  ( $5 \times 10.66 = 53.3$ ), and the long period of  $100 \pm 15$  yr may be related to the known Gleissberg period of 87 yr.



**Figure 1.** The solar cycle length (SCL) from 1610 as downloaded from the National Geophysical Data Center (NGDC). We observe that the SCL was longer than the mean of 11.06 yr in most of the 19th century and shorter than the mean in most in the 20th century.

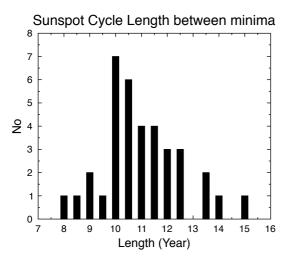


Figure 2. The distribution of the solar cycle lengths in bins of 0.5 yr width. The distribution covers 36 cycles from 1610 to 2008.

# 3.2 Long-term modulation of the length of the solar cycle

We use the average period P = 11.06 yr as our reference period and obtain the O–C residuals as shown in Fig. 4, where the O–C residuals are given as a function of the cycle no. As the starting point for cycle -13 we use 1610.8 with an O–C = -0.95. The residuals give us a picture of the long-term trends in SCL. We observe that the residuals increase most of the time between SC4 and SC14 (1775–1900), because the SCL is then nearly always longer than 11 yr (see also Fig. 1). Then we enter a period with shorter periods, and a warming Earth. The question is now if that will continue.

To investigate what controls the length of the solar cycle, we calculate a periodogram of the residual O–C data string, and get the amplitude spectrum shown in Fig. 5.

The spectrum consists of two dominating periods:  $190 \pm 9$  and  $85.6 \pm 2$  yr. Periods shorter than 50 years are harmonics of the two main periods. There is also a period of the order 440 yr, which explains why the peak around 1900 is higher

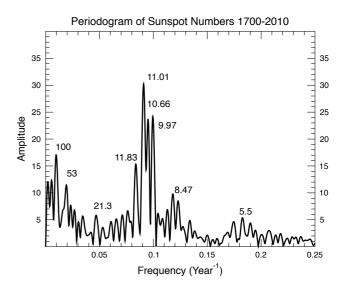


Figure 3. Amplitude spectrum of the yearly average sunspot numbers 1700–2010.

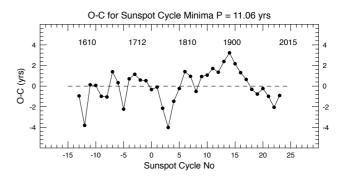


Figure 4. O–C residuals for the length of the solar cycle compared with the average period of 11.06 yr. The curve is increasing for SCL > 11.06 yr.

than the peak around 1700. A similar result was obtained by Richards et al. (2009), who identified a Gleissberg period of  $86.5\pm12.5$  yr and a secular period of  $188\pm38$  yr. In their analysis they use SCLs based on both solar maxima and minima.

In Fig. 6 we show the O–C residuals with the strongest controlling period  $\approx 190\,\mathrm{yr}$  and its subharmonic at  $\approx 440\,\mathrm{yr}$ . This dominant cycle is the reason for an increasing period length in the 19th century and a decreasing length in the 20th century. We can therefore expect increasing SCLs in the 21st century.

Adding the Gleissberg cycle and three of the harmonics gives the fit shown in Fig. 7, where we may also obtain an estimate of near future SCLs. Times of minima can be estimated from the following equation:

$$t_{\min} = 1755.5 + 11.06 \times N_i + (O-C)_{\text{est}},$$
 (2)

where (O–C)<sub>est</sub> is the estimated O–C value determined with the harmonic model as shown in Fig. 7 (red curve). For the

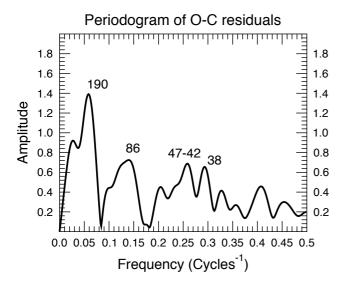


Figure 5. Amplitude spectrum of O–C residuals of the SCL measured between minima.

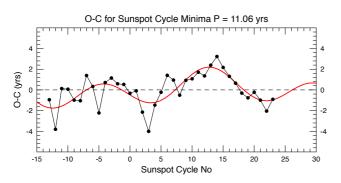


Figure 6. O–C residuals for SCL minima, with a simulation based on the dominating periods of 190 and 440 yr.

next minimum after SC24, Eq. (2) gives 2020.9, since the  $(O-C)_{est}$  then is close to zero.

### 4 Discussion

We have shown that the solar cycle length since 1600 is controlled by stable oscillations, which provide an average cycle length of 11.06 yr. The cycle length is modulated by 3 long periods of  $\approx$  440,  $\approx$  190 and  $\approx$  86 yr, and some of their harmonics. If the dominating period of  $\approx$  190 yr is followed back in time, it is found (Richards et al., 2009) that all known solar deep minima during the last 1000 yr (the Oort, Wolf, Spörer, Maunder and Dalton minima) are close to the minimum or on the rising branch of this oscillation. We can therefore expect another grand minimum during the first part of this century.

Looking more closely at the model simulations in Fig. 7, we estimate the length of SC24  $\approx$  12 yr, SCL25  $\approx$  9 yr, SCL26  $\approx$  11 yr and SCL27  $\approx$  14 yr. The forecast for the time of the next minimum (2020.9) can be compared with the forecast

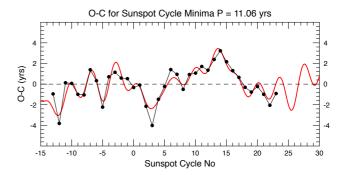


Figure 7. O–C residuals for SCL minima, with a simulation based on 6 harmonics with periods 440, 190, 86, 48, 43, and 38 yr.

based on a mathematical model (Salvador, 2013), which estimates the end of solar cycle 24 in 2018.

It has for some time been discussed if the solar cycle length is controlled by an internal or external clock. Dicke (1978) argued that the phase of the solar cycle appears to be coupled to an internal clock, because shorter cycles are usually followed by longer cycles, as if the Sun remembers the correct phase. Another view (Huyong, 1996) is that the memory effect can be explained by mean field theory, which predicts coherent changes in frequency and amplitude of a dynamo wave. However, it is admitted by solar physicists that present solar dynamo theories, although able to describe the periodicities and the polarity reversal of solar activity well, are not yet able to quantitatively explain the 11 and 22 yr cycles, nor the other observed quasi-cycles (de Jager and Versteegh, 2005). The remarkable resemblance between planetary tidal forcing periods and observed solar quasi-periods is a strong argument for a planetary tidal forcing on the solar activity.

Regarding the splitting of the 11 yr solar cycle band into 4 distinct peaks, the most remarkable is the strongest peak  $P = 11.010 \pm 0.001$  yr. A period so close to 11 Earth years has a great chance to be related to the Earth's orbit. Wilson (2013) explains that the Venus–Earth–Sun periodic alignments create a tidal bulge, which for a period of 11.07 yr is speeded up by Jupiter's movement, and the next 11.07 yr are slowed down by the same. This is called the VEJ tidal-torque coupling model, and explains both the average Schwabe and Hales cycles. These tidal forces work to increase or decrease the solar rotation rate in the convective layers where the solar dynamo is situated (Wilson, 2013).

Among the other three periods in the 11 yr band, 9.97 yr is close to the Jupiter–Saturn spring tide period of 9.93 yr, which is half of the Jupiter–Saturn heliocentric conjunction period of 19.86 yr. It should be noticed that the spring tide period of Jupiter/Saturn varies between 9.5 and 10.5 yr (Scafetta, 2012). The period of 11.83 yr is close to Jupiter's orbital period of 11.86 yr. Scafetta (2012) proposes that the solar cycle period  $\approx$  11.0 yr is generated by the two side attractors controlled by the two giant planets. We have found another sunspot period at 10.66 yr, which also may be a

dynamo period. Both these periods are strongly forced, since they have higher harmonics of 5.5 and 5.25 yr, and one subharmonic of 21.3 yr.

By our O–C analysis we find, as did Richards et al. (2009), that the SCL is modulated by a secular period of  $190 \pm 9$  yr in addition to a period of  $86 \pm 2$  yr, which most likely is the Gleissberg period. The long period is close to the Jose cycle of 178.7 yr, which is the period of recurrent pattern of the movement of the Sun around the barycenter of the solar system (Jose, 1965). Fairbridge and Hameed (1985) found phase coherence of solar cycle minima over two 176 yr cycles, or 16 Schwabe periods. Our 190 yr period is also close to a period of 208 yr, which is found in cosmic ray observations and in cosmogenic isotopes, and explained by tidal torque on the Sun by the planets (Abreu et al., 2012).

However, a far better match with the 190 yr period is found by introducing a so-called Gear Effect, which modulates the tangential torque applied by the alignments of Venus and Earth to the Jupiter-Sun-Saturn system as explained by Wilson (2013). He shows that prograde and retrograde torque oscillate in a quasi bidecadal period controlled by the 19.859 yr synodic period of Jupiter and Saturn. Figure 13 in Wilson (2013) shows the angel between the center of mass of the Jupiter, Sun and Saturn system and Venus/Earth from 1013 to 2015. If we compare this with our Fig. 6, we find an excellent match between periods and phases, indicating a strong link between the modulation of the solar cycle length and the torque effect proposed by Wilson (2013). The modulation period can be calculated as the beat period between the Hale-like period of 22.137 yr and the Jupiter–Saturn synodic period of 19.859 yr. The result is a beat period of 192.98 yr or  $193 \pm 2$  yr, when the orbital variations are included (Wilson, 2013). By also introducing the Gear Effect to the VEJ-tidal torque model, he can also explain an 88.1 yr Gleissberg cy-

Finally, it may be instructive to compare our predictions of the next solar cycle lengths with a prediction made by de Jager and Duhau (2009), based on the dynamo model that is constructed from the relationship between the polodial and torodial magnetic cycles. They conclude that the polar cycle 24 will be similar to polar cycle 12, which means that the maxima of sunspot cycles 23 and 24 will be quite similar to those of the cycle pair 11 and 12. They further conclude that a short Dalton minimum will occur, lasting a maximum of 3 cycles (SC24-26), whereafter a grand minimum will follow, starting with cycle 27. They predict the maximum sunspots of SC24 to be  $68 \pm 17$  with a maximum at  $2014.5 \pm 0.5$ , but do not predict the length.

At the moment we are close to the solar maximum of SC24, but have 7 more years to the next minimum, according to our forecast. During that period we will observe if the cooling forecast for the North Atlantic region will take place, and if this will also keep the global temperature in hiatus, as it has been since the start of SC23.

#### 5 Conclusions

We have shown that the Schwabe frequency band of the sunspot record since 1700 has an average period of 11.06 yr and contains four major cycles, with periods of 9.97, 10.66, 11.01 and 11.83 yr. Analysis of the O–C residuals of the timing of solar cycle minima reveals that the solar cycle length is modulated by a secular period of about 190 yr and a Gleissberg period of about 86 yr. Our result is a confirmation of earlier phase studies by Fairbridge and Hameed (1983) and Richards et al. (2009).

Based on a simple harmonic model with these periods, we predict that the solar cycle length will increase during the 21st century. Cycle 24 may be about 12 yr long, while cycles 25 and 26 are estimated to be about 9 and 11 yr long. The following cycle 27 will be much longer. In all periods when the solar cycle length has increased due to the 190 yr cycle during the last 1000 yr, a deep minimum of solar activity has occurred. This is also to be expected in the early part of this.

The coherent modulation of the solar cycle length over a period of 400 yr is a strong argument for an external forcing by the planets Venus, Earth, Jupiter and Saturn, expressed in the spin-orbit coupling model as proposed by Wilson (2013).

Excellent phase coherence with this model is a strong added argument for this interpretation.

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