Pattern Recogn. Phys., 1, 117–122, 2013
www.pattern-recogn-phys.net/1/117/2013/
doi:10.5194/prp-1-117-2013

Pattern Recognition
in Physics
Open Access

A mathematical model of the sunspot cycle for the past 1000 yr

R. J. Salvador

Vancouver, Canada

Correspondence to: R. J. Salvador (rj_salvador@hotmail.com)

Received: 8 October 2013 – Revised: 27 October 2013 – Accepted: 30 October 2013 – Published: 15 November 2013

Abstract. Using many features of Ian Wilson's Tidal Torque theory, a mathematical model of the sunspot cycle has been created that reproduces changing sunspot cycle lengths and has an 85 % correlation with the sunspot numbers from 1749 to 2013. The model makes a reasonable representation of the sunspot cycle for the past 1000 yr, placing all the solar minimums in their right time periods. More importantly, I believe the model can be used to forecast future solar cycles quantitatively for 30 yr and directionally for 100 yr. The forecast is for a solar minimum and quiet Sun for the next 30 to 100 yr. The model is a slowly changing chaotic system with patterns that are never repeated in exactly the same way. Inferences as to the causes of the sunspot cycle patterns can be made by looking at the model's terms and relating them to aspects of the Tidal Torque theory and, possibly, Jovian magnetic field interactions.

1 Introduction

Considerable evidence now exists that the Earth's climate is heavily dependent on the solar cycle. The forecasting of solar cycles has mainly concentrated on predicting the future course of the present or the next cycle. Longer range and accurate predictions of the solar cycle pattern are necessary for understanding the future course of the Earth's climate. A useful model of the solar cycle should be able to reconstruct or recast historical solar cycles from proxy reconstructions as well as the modern record to have any credibility. Models based on theories of the dynamics of the solar dynamo are unable to do this. However, theories based on perturbations to the solar dynamo based on planetary interactions with the sun show more promise. In a recent publication, Scafetta (2012) discusses the state of solar forecasting and proposes a simplified solar cycle model based on three harmonics found in the power spectrum of the sunspot number record. Scafetta suggests that the solar cycle can be characterized by constructive and destructive interference patterns. His model successfully reconstructs the timing and pattern of past solar minimums in generic units and forecasts a solar minimum in the 2020–2045 time frame.

The model presented here is an attempt to produce a more quantitative prediction of monthly sunspot number forecasts and increase the granularity of the shape of future solar cycles. The model is based primarily on a Tidal Torque theory proposed by Wilson (2011) and two Jovian harmonics that account for the positioning of three Jovian planets.

Wilson's theory proposes that periodic alignments of Venus and the Earth on the same or opposite sides of the Sun produce temporary solar tidal bulges. Jupiter's gravitational force acts on these bulges and either speeds up or slows down the rotation of the Sun's plasma, leading to changes in solar activity. The frequency of these alignments on the same side of the Sun is 22.14 yr. Wilson also shows that the strength of the tidal force depends on the heliocentric latitude of Venus and the mean distance of Jupiter from the Sun, and that when these forces are weakest, solar minimums occur. This happens approximately every 165.5 yr. The frequency to produce a 165.5 yr beat with 22.14 yr is 19.528 yr. These two frequencies of Venus–Earth–Jupiter (VEJ) interactions are a principle basis for the model.

Wilson et al. (2008) have also shown the connection between the Hale cycle (22.1 ± 1.9 yr) and the synodic period of Jupiter and Saturn (19.859) such that their beat frequency is 178.8 yr, which is the Jose cycle. The Jupiter–Saturn synodic and the Jose cycle frequency are used in the model.

Sharp (2013) has proposed a connection between the Uranus and Neptune synodic and grand solar minimums. Sharp (http://thetempestspark, 2013) has also produced a very instructive animation of the odd polar orientation and orbital pattern of Uranus and graphics showing how the planet's polar orientation aligns with individual solar cycle minimums. The one-quarter Uranus orbital frequency of 21.005 is used in the model.

Another well-known oscillation found in solar records is the de Vries cycle of 208 yr (see McCracken et al., 2013). The frequency of 1253 yr, together with the Jose frequency of 178.8 yr, produces a beat of 208 yr and is used in the model.

2 The model

This model is simply four interacting waves, but they are modulated to create an infinite possibility for sunspot formation.

The basic frequencies in years are:

– a VEJ frequency of 22.14 (varying),

– a VEJ frequency of 19.528 (varying and forming a beat frequency of 165.5 with 22.14),

– Jupiter–Saturn synodic frequency of 19.858,

– one-quarter Uranus orbital frequency equal to 21.005,

– two modulating frequencies of 178.8 and 1253 (forming a beat frequency of 208 yr).

Individual sunspot cycles have varying cycle lengths and this is an impediment to obtaining a continuous mathematical model for correlation. The monthly sunspot data imply that frequencies and/or phasing of the basic cycles are slowly changing over time.

It should be noted that the 178.8 frequency is also the time of rotation of the Sun around the barycenter. The perturbations described by the VEJ and Jovian frequencies are in the Sun, and therefore it is plausible that solar acceleration reasons could cause modulations to these frequencies (see Cionco and Campagnucci, 2012). This provided the idea that perhaps the Jovian 19.858 and 21.005 and the VEJ 22.14 frequencies and phases are changing over time to the barycenter rotation of 178.8.

During this work it was also found that the 19.528 VEJ frequency is changing to the slower 1253 frequency. Likewise there is a possible explanation for this in the time it takes for the orbital realignment of Venus, the Earth and Jupiter to return to the same position against the stars. See Wilson (2013 Hallstatt).

These frequency- and phase-changing capabilities are built into the model and for the most part solve the cycle length problem for correlation.

The model does not reproduce the skewed Gaussian shape of the sunspot cycles, as the model attempts to simulate the forces activating the cycle, and not the process of actual sunspot formation and disappearance. Since the time length of the formation of sunspots is unstated, the phasing in the model is left open and determined by correlation.

The following is the mathematical construction of the model.

The sunspot data was transformed into positive and negative oscillations by multiplying the monthly sunspot number (SN) by the sunspot cycle's polarity of plus or minus one.

$$\mathrm{SNC} = \mathrm{SN} \times \mathrm{POLARITY}$$

The data was then correlated to the following equation, where the Fs and Ns are scalars and the Ls and Ps are phasing parameters, and all are determined by a non-linear least squares optimization:

$$\begin{aligned}\mathrm{SNC} = {} & (F1 \times \cos(w1 \times (t + \mathrm{ph1})) + F2 \times \cos(w2 \times (t + \mathrm{ph2})) \\ & + F3 \times \cos(w3 \times (t + \mathrm{ph3})) + F4 \times \cos(w4 \times (t + \mathrm{ph4}))\end{aligned}$$

SNC is the polarity-adjusted sunspot number and T is the time in calendar years.

Ws are the modulated frequencies and are changed by either 178.8 or 1253.

$$w1 = 2 \times pi/(19.528 \times (1 + N1 \times \cos(2 \times pi/1253 \times (t + L1))))$$

$$w2 = 2 \times pi/(22.14 \times (1 + N3 \times \sin(2 \times pi/178.8 \times (t + L2))))$$

$$w3 = 2 \times pi/(19.858 \times (1 + N5 \times \cos(2 \times pi/178.8 \times (t + L3))))$$

$$w4 = 2 \times pi/(21.005 \times (1 + N7 \times \sin(2 \times pi/178.8 \times (t + L4))))$$

Phs are the modulated phases of each component of the model and are changed by the frequency of 178.8 or 1253.

$$\mathrm{ph1} = P1 \times (1 + N2 \times \cos(2 \times pi/1253 \times (t + L1)))$$

$$\mathrm{ph2} = P2 \times (1 + N4 \times \sin(2 \times pi/178.8 \times (t + L2)))$$

$$\mathrm{ph3} = P3 \times (1 + N6 \times \cos(2 \times pi/178.8 \times (t + L3)))$$

$$\mathrm{ph4} = P4 \times (1 + N8 \times \sin(2 \times pi/178.8 \times (t + L4)))$$

A model with frequencies of 1253 and 178.8 cannot be properly calibrated with only 300 yr of monthly sunspot data, as this covers only 20 % of the 1253 cycle and only one and a half cycles of the 178.8 frequency. To overcome this difficulty, sunspot data over a much longer time period are needed. Solanki et al. (2004) have reconstructed ten year average sunspot numbers for the past 11 000 yr from available ^{14}C records. Since the model requires monthly data (not 10 yr averages) and the polarity of the cycle, the Solanki data (2005) cannot be used in total. However, the Solanki data does quantify three time periods in the past 1000 yr when the sunspot number was zero, viz, the Maunder, Spörer and Wolfe minima. These monthly time periods, as defined by Solanki, can be used with the sunspot number set to zero, and then the polarity becomes a non-issue. Figure 1 shows the Solanki data (Solanki et al., 2004; Solanki, 2005) from

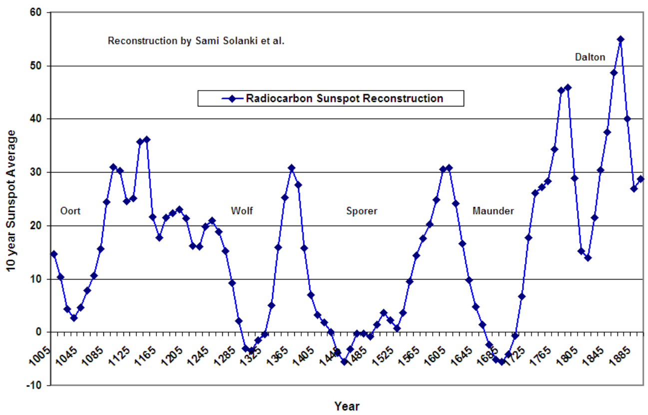

Figure 1. Solanki et al. (2004) reconstruction data from the years 1000 to 1895 of 10 yr average sunspot numbers from radio carbon 14 data, showing time periods when the average sunspot number was at or near zero.

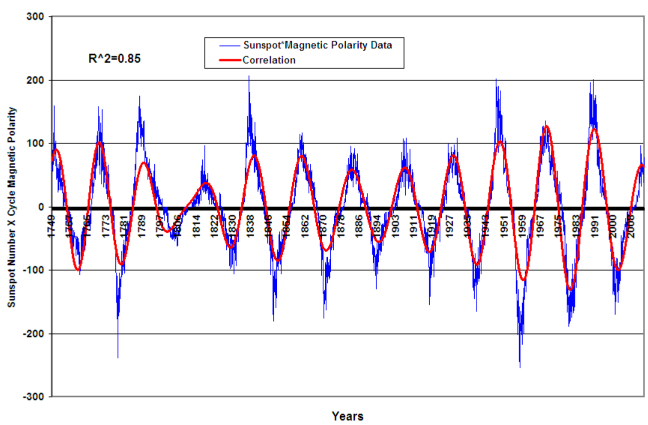

Figure 2. A comparison of the cyclical oscillation of the monthly sunspot number multiplied by a cycle polarity of plus 1 for even cycles and −1 for odd cycles from 1749 to 2013 (in blue) with the correlation model (in red).

the years 1000 to 1895, where it stops due to interferences by activities of modern society.

Using this additional data, the model has a strong correlation of $R^2 = 0.85$ for the data between 1749 and 2013, and produces a very interesting and reasonable reproduction of sunspot cycles for the past 1000 yr. Figure 2 illustrates the 1749–2013 correlation as a cyclical oscillation and Fig. 3 shows the same result in the more usual absolute value form.

3 Sunspot reconstruction for the last 1000 yr

In Fig. 4, the model is used to reconstruct the sunspot cycles from the year 1000 to the present and compared to the sunspot average data set of Solanki et al. (2004), to which the number 40 has been added to each data point to better illustrate the correspondence of the Solanki averages to the model's monthly sunspot number. This 1000 yr correlation model constitutes the basis for forecasting.

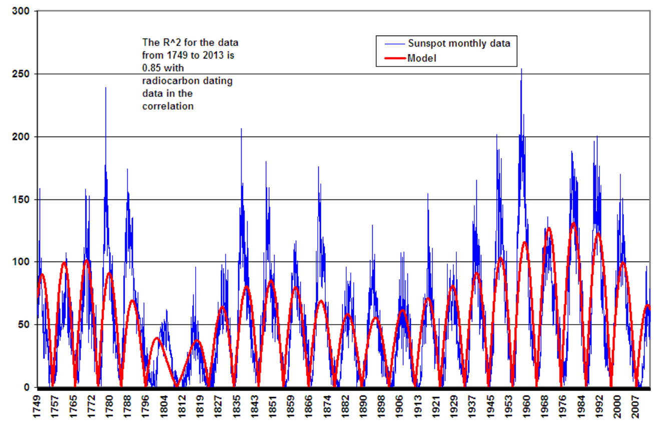

Figure 3. A comparison of monthly sunspot numbers from 1749 to 2013 (in blue) with the absolute value of the correlation model (in red), derived using the observational data from 1749 to 2013 and the additional data from Solanki et al.

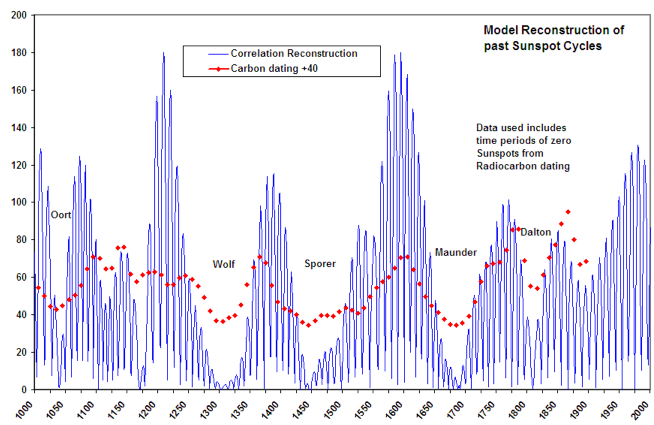

Figure 4. A comparison of monthly sunspot numbers from 1000 to 2013 (in blue), calculated from the absolute value of the correlation model, with the Solanki et al. (2004) average-derived sunspot numbers. 40 units are added to the Solanki data for illustrative purposes.

4 Forecasting

To test if the model has forecasting ability, we can redo the correlation with data only up to the years 1950 and 1900 and determine the forecast for the next 50 and 100 yr to see if the model can predict the sunspot data we have already experienced.

Figure 5 gives a forecast for the period 1950 to 2050 made from the correlation of the model with data up to 1950. The model forecasts a peaking sunspot cycle and a significant decline in sunspots around the turn of the century, and an ongoing solar minimum. The model is a little early, but directionally correct 50 yr out.

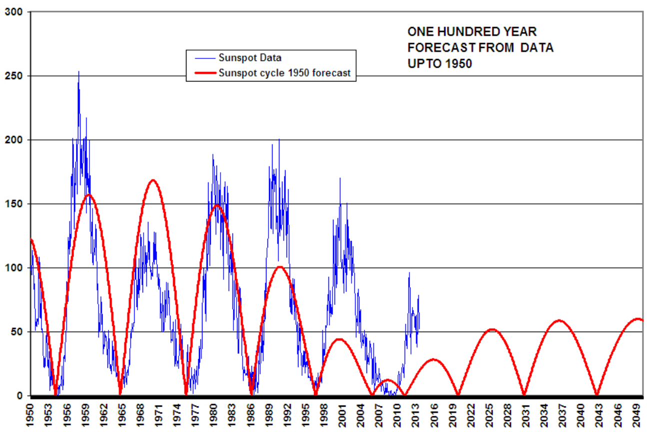

Figure 5. A comparison of monthly sunspot numbers from 1950 to 2013 (in blue) with the absolute value of the correlation model (in red), derived using data only up to 1950 and the extended forecast to 2050.

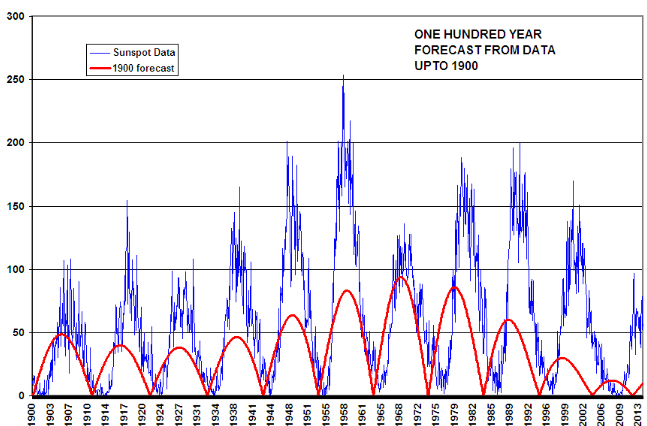

Figure 6. A comparison of monthly sunspot numbers from 1900 to 2000 (in blue) with the absolute value of the correlation model (in red), derived using data only up to 1900 and the extended forecast to 2000.

Figure 6 gives a similar forecast made with data up to 1900.

Although the model did not predict the magnitude of the increase in spot activity 50 yr past 1900, it did forecast increasing and then decreasing sunspot activity, with a minimum around the turn of the century.

I believe this shows the model has credibility in forecasting two to three sunspot cycles out and directionally for one hundred years.

Figure 7 gives a forecast made with data up to 2013. The forecast predicts a very quiet Sun for the next 100 yr. The model forecasts that the sunspot cycle will not produce sunspot values over 100 again until the cycle that starts around 2160; however, that is beyond the usable time horizon of this model.

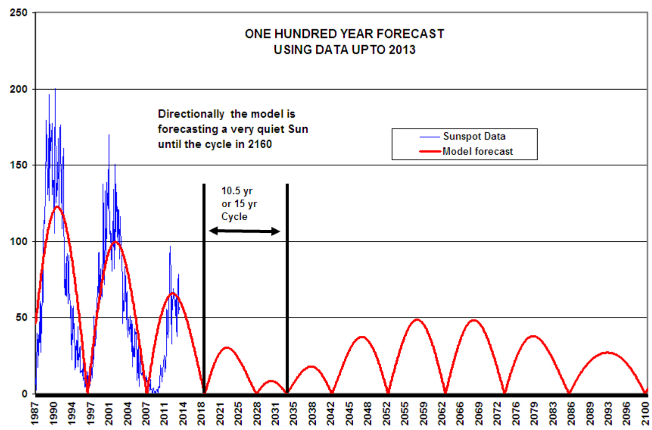

Figure 7. A comparison of monthly sunspot numbers from 1987 to 2013 (in blue) with the absolute value of the correlation model (in red), derived using data up to 2013 and the extended forecast to 2100.

The model forecasts that the existing cycle, 24, will end in 2018. The next cycle, 25, could prove to be very interesting, as the model predicts it will be difficult to tell when it ends and the next one begins. The duration of cycle 25 will be either 10.5 or 15 yr long. The model forecasts that a pronounced grand solar minimum will persist from the start of cycle 25 in 2018 out to 2060. The 100 yr, multi-cycle prediction, which shows a small rise then a further decline in cycle magnitude, suggests the minimum may extend beyond 2060. The forecast for a grand minimum in this time period is consistent with the predictions of Mörner (2011), Scafetta (2012), (2013) and Cionco and Compagnucci (2012).

5 Sunspot activation

It is instructive to examine the model for the destructive and constructive wave interactions that produce a Maunder minimum (Fig. 8) or a modern maximum, to determine if there are some implications as to how the solar system may be affecting sunspot cycles.

Figure 9 gives the sum of the two terms of the VEJ cycle (19.528 and 22.14) and the two terms of the Jovian cycles (19.585 and 21.005) from the years 1600 to 2200. The model gives equal weight in magnitude to the VEJ and Jovian cycles. These cycles can hide, and interfere both constructively and destructively with each other.

The model's two interference patterns, in turn, interfere with each other to produce the minima and maxima of the solar cycles. For example, the Dalton minimum occurred at a minimum in both the VEJ and Jovian cycles. Yet the Maunder minimum resulted from destructive wave interference when both cycles were near maxima. The Modern maximum is a result of constructive interference from a maximum in both cycles. The coming solar minimum is the result of wave pattern destructive interference between the VEJ and Jovian

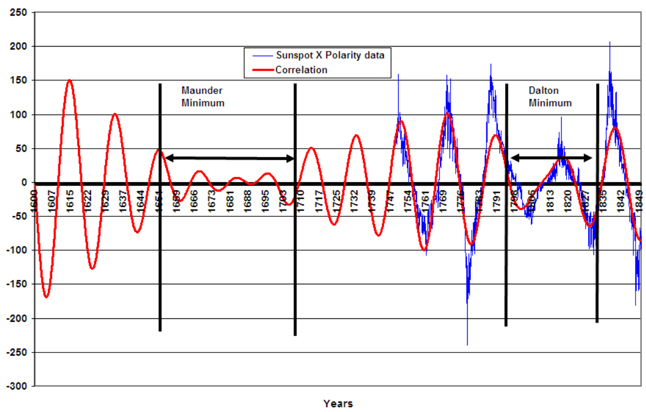

Figure 8. The red line is the model's reconstruction of the cyclical oscillation of the monthly sunspot number multiplied by a cycle polarity of plus 1 for even cycles and −1 for odd cycles from 1600 to 1850, which include the Maunder and Dalton solar minimums. The actual sunspot data multiplied by the cycle polarity for the monthly time periods from 1749 to 1850 is in blue.

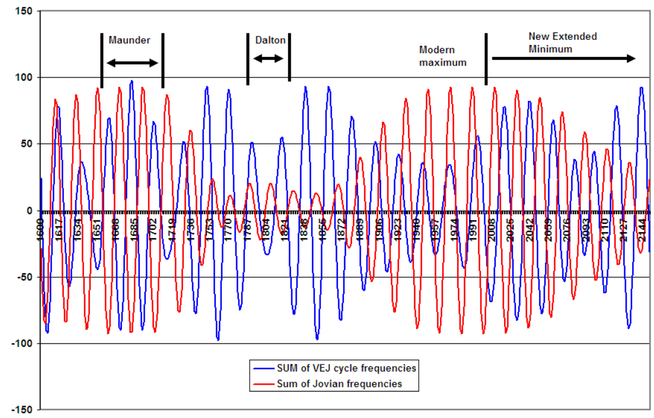

Figure 9. The blue line is the interference contribution pattern for the sum of the two VEJ frequencies (19.528, 22.14), and the red line is the interference contribution for the sum of two Jovian frequencies (19.585, 21.005) to the polarity-adjusted sunspot model for the years 1600 to 2100. The periods of destructive interference during solar minimums and constructive interference during the solar maximum can be seen by inspection of these two interference patterns. At times either the VEJ or Jovian cycles can dominate.

cycles and is extended by minimized VEJ and Jovian internal destructive interference.

This model will not work without the influence of the Uranus one-quarter orbital frequency of 21.005. The unusual orbital rotation of Uranus around its equator, I believe, is a possible indication of a magnetic to magnetic field interaction.

The VEJ and Jovian oscillations change through time, so that the same precise pattern never repeats itself. At present the VEJ cycle has an oscillation of 165.5 yr and the Jovian cycle 363.6 yr, but these change as the base frequencies and phasing are modulated.

6 Conclusions

The model predicts that the sun is entering a grand minimum, and the general shape of the model's future multi-cycle projections suggests that this minimum may persist for an extended period of time.

I believe this model captures a fundamental relationship between a gravitational disturbance in the Sun's magnetic field through the Tidal Torque process and a magnetic disturbance in the Sun's magnetic field through the Jovian planets.

I also believe this model describes a chaotic process where small changes in frequency and/or phase modulation parameters over time lead to large variations in individual solar cycle outcomes. Fortunately because the changes to the base frequencies and phasing occur slowly in terms of human life spans, we can make forecasts that may be useful.

7 Model constants

The model can be constructed in an Excel spreadsheet using the equations in this article and the values can be provided by the author through contact at this e-mail address: (rj_salvador@hotmail.com).

Acknowledgements. The author wants to express his thanks to Ian Wilson for the Tidal Torque theory and frequencies, Paul Vaughan for laying out in detail the relationship between frequencies, Tim Channon for a very timely comment, and Roger Tattersall for pointing out the Uranus one-quarter frequency and helpful articles.

Edited by: N.-A. Mörner
Reviewed by: H. Jelbring and one anonymous referee

References

Cionco, R. G. and Compagnucci, R. H.: Dynamical characterization of the last prolonged solar minima, Adv. Space Res., 50, 1434–1444, 2012.

McCracken, K. G., Beer, J., Steinhilber, F., and Abreu, J.: A phenomenological study of the cosmic ray variations over the past 9400 years, and their implications regarding solar activity and the solar dynamo, Sol. Phys., 286, 609–627, 2013.

Mörner, N-A.: Arctic environment by the middle of this century, Energy & Environment, 22, 207–218, 2011.

NASA, Sunspot Monthly Data, http://solarscience.msfc.nasa.gov/greenwch/spot_num.txt, 2012.

Scafetta, N.: Multi-scale harmonic model for solar and climate cyclical variation throughout the Holocene based on

Jupiter–Saturn tidal frequencies plus the 11-year solar dynamo cycle, J. Atmos. Sol.-Terr. Phy., 80, 296–311, 2012.

Scafetta, N.: Solar and planetary oscillation control on climate change: hindcast, forecast and a comparison with the CMIP5 GCMs, Energy & Environment, 24, 455–496, 2013.

Sharp, G. J.: Are Uranus & Neptune Responsible for Solar Grand Minima and Solar Cycle Modulation?, International Journal of Astronomy and Astrophysics, 3, 260–273, 2013.

Sharp, G. J.: http://thetempestspark.files.wordpress.com/2013/06/3-0.gif, last access: 12 November 2013.

Sharp, G. J.: http://thetempestspark.files.wordpress.com/2013/09/uranus-solar-4-rbg2.gif, last access: 12 November 2013.

Solanki, S. K.: 11,000 year sunspot reconstruction, NOAA Paleoclimatology Program and World Data Centre for Paleoclimatology, ftp://ftp.ncdc.noaa.gov/pub/data/paleo/climate_forcing/solar_variability/solanki2004-ssn.txt, 2005.

Solanki, S. K., Usoskin, I. G., Kromer, B., Schüssler, M., and Beer, J.: An unusually active Sun during recent decades compared to the previous 11,000 years, Nature, 431, 1084–1087, 2004.

Wilson, I.: Tidal-Torque model of Solar-Planetary interaction, posted 12 November, http://tallbloke.wordpress.com, 2012.

Wilson, I. R. G.: Do periodic peaks in the planetary tidal forces acting upon the Sun influence the sunspot cycle?, The General Science Journal, Dec 2011, 3812, 2011.

Wilson, I. R. G.: The VEJ Tidal Torquing model can explain many of the long-term changes in the level of solar activity, II. The 2300 year Hallstatt Cycle, posted 12 August, http://astroclimateconnection.blogspot.com.au/2013/08/the-vej-tidal-torquing-model-can.html, 2013.

Wilson, I. R. G., Carter, B. D., and Waite, I. A.: Does a spin-orbit coupling between the Sun and the Jovian Planets govern the solar cycle?, Publ. Astron. Soc. Aust., 25, 85–93, 2008.